PROPOSTAS DE RESOLUÇÃO

Capítulo 8

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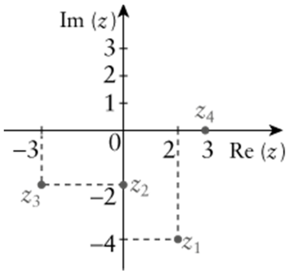
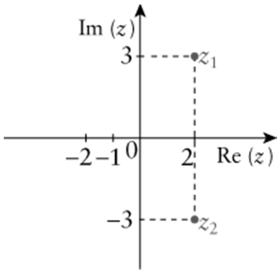
1.1.

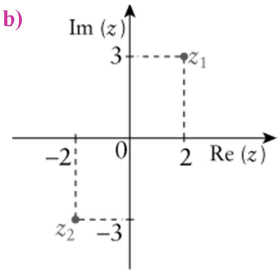


1.2. **a)**



Observação: As imagens geométricas de z_1 e z_2 são simétricas relativamente ao eixo real.

b)



Observação: As imagens geométricas de z_1 e z_2 são simétricas relativamente à origem do referencial.

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2.1. **a)** $(2-3i)+(4-i)=$

$=2-3i+4-i=6-4i$

b) $(-1+i)-(2-3i)=-1+i-2+3i=$

$=-3+4i$

c) $(2+i)-3i=2+i-3i=2-2i$

d) $\left(\frac{3}{2}+i\right)-\left(\frac{3}{4}-i\right)=\frac{3}{2}+i-\frac{3}{4}+i=$

$=\frac{6}{4}-\frac{3}{4}+2i=\frac{3}{4}+2i$

e) $\left(\frac{1}{2}-2i\right)+\left(-1+\frac{1}{2}i\right)=\frac{1}{2}-2i-1+\frac{1}{2}i=$

$=\frac{1}{2}-\frac{2}{2}-\frac{4}{2}i+\frac{1}{2}i=-\frac{1}{2}-\frac{3}{2}i$

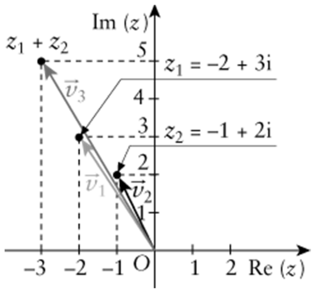
2.2. $z_1+z_2=(a+bi)+(c+di)=a+bi+c+di$

$=(a+c)+(b+d)\,i$

2.3. **a)** $z_1+z_2=(-2+3i)+(-1+2i)$

$=-2+3i-1+2i$

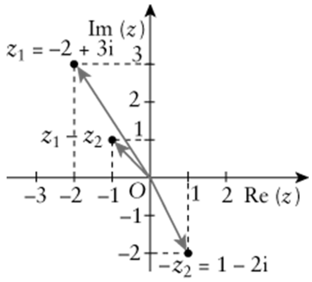
$=-3+5i$



b) $z_1-z_2=(-2+3i)-(-1+2i)$

$=-2+3i+1-2i$

$=-1+i$



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3.1. **a)** $(2+i)i=2i+i^2$

$=2i+(-1)=2i-1=-1+2i$

b) $i(1-5i)=i-5i^2=i-5(-1)=i+5=5+i$

c) $(2-i)\left(3+\frac{5}{2}i\right)=6+\frac{10}{2}i-3i-\frac{5}{2}i^2$

$=6+5i-3i-\frac{5}{2}(-1)=6+2i+\frac{5}{2}$

$=\frac{12}{2}+\frac{5}{2}+2i=\frac{17}{2}+2i$

d) $\left(-3+i\right)\left(-\frac{1}{2}-i\right)=\frac{3}{2}+3i-\frac{1}{2}i-i^2$

$=\frac{3}{2}+\frac{6}{2}i-\frac{1}{2}i-(-1)=\frac{3}{2}+\frac{5}{2}i+1$

$=\frac{5}{2}+\frac{5}{2}i$

3.2. $(a+bi)(c+di)=ac+adi+bci+bdi^2$

$=ac+adi+bci-bd=(ac-bd)+(ad+bc)i$

4.1. $(2-3i)(2+3i)=2^2-(3i)^2=4-9i^2=4+9=13$

4.2. $(1-i)(1+i)=1^2-i^2=1+1=2$

4.3. $(-1+i)(i+1)=(i-1)(i+1)=i^2-1^2$

$=-1-1=-2$

5.1. $(-2+3i)^2=4-12i+9^2=4-12i-9=-5-12i$

5.2. $\left(-1-\frac{1}{2}i\right)^2=1+i+\frac{1}{4}i^2=1+i-\frac{1}{4}i=\frac{3}{4}+i$

5.3. $(-1+i)^3=(-1+i)^2(-1+i)$

$=(1-2i+i^2)(-1+i)=(1-2i-1)(-1+i)$

$=(-2i)(-1+i)=2i-2i^2=2+2i$

5.4. $(a+bi)^2=a^2+2abi+(bi)^2=a^2+2abi+b^2i^2$

$=a^2+2abi-b^2=(a^2-b^2)+2\,abi$

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6.1. $\frac{1-2i}{i}=\frac{(1-2i)(-i)}{i\times(-i)}=\frac{-i+2i^2}{-i^2}$

$=\frac{-1-2}{1}=-2-i$

6.2. $\frac{1-2i}{1+i}=\frac{(1-3i)(1-i)}{(1+i)(1-i)}=\frac{1-i-3i+3i^2}{1^2-i^2}$

$=\frac{1-4i-3}{1+1}=\frac{-2-4i}{2}=-1-2i$

6.3. $\frac{3}{-1+2i}=\frac{3(-1-2i)}{(-1+2i)(-1-2i)}$

$=\frac{-3-6i}{(-1)^2-(2i)^2}=\frac{-3-6i}{1-4i^2}$

$=\frac{-3-6i}{5}=-\frac{3}{5}-\frac{6}{5}i$

6.4. $(2-3i)^{-1}=\frac{1}{2-3i}=\frac{1\times(2+3i)}{(2-3i)(2+3i)}$

$=\frac{2+3i}{4-9i^2}=\frac{2+3i}{13}=\frac{2}{13}+\frac{3}{13}i$

6.5. $\frac{2+3i}{z}=1-5i\Leftrightarrow\frac{2+3i}{1-5i}=z$

$\Leftrightarrow\frac{(2+3i)(1+5i)}{(1-5i)(1+5i)}=z$

$\Leftrightarrow\frac{2+10i+3i+15i^2}{1-25i^2}=z$

$\Leftrightarrow\frac{2+13i-15}{1+25}=z\Leftrightarrow\frac{-13+13i}{26}=z$

$\Leftrightarrow-\frac{1}{2}+\frac{1}{2}i=z$

6.6. $\frac{a+bi}{c+di}=\frac{(a+bi)(c-di)}{(c+di)(c-di)}=\frac{ac-adi+bci-bdi^2}{c^2-d^2i^2}$

$=\frac{ac-adi+bci+bd}{c^2+d^2}=\frac{ac+bd}{c^2+d^2}+\frac{bc-ad}{c^2+d^2}i$

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7.1. $i^{39}=(i^4)^9\times i^3=i^3=-i$

Cálculo auxiliar $\begin{array}{r|l} 39 & 4 \\ \hline 03 & 9 \end{array}$

7.2. $i^{37}+i^{999}-(2i)^{25}=i^{37}+i^{999}-2^{25}i^{25}$

$=(i^4)^9\times i+(i^4)^{249}\times i^3-2^{25}(i^4)^6\,i$

$=i+i^3-2^{25}\,i$

$=i-i-2^{25}\,i$

$=-2^{25}\,i$

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8. Consideremos que $a+bi$ é uma raiz quadrada de $-3+4i$.

Então:

$-3+4i=(a+bi)^2\Leftrightarrow-3+4i=a^2+2abi-b^2$

$\Leftrightarrow\begin{cases}-3=a^2-b^2\\2ab=4\end{cases}\Leftrightarrow\begin{cases}-3=a^2-\left(\frac{2}{a}\right)^2\\b=\frac{2}{a},a\neq0\end{cases}$

$\Leftrightarrow\begin{cases}-3=a^2-\frac{4}{a^2}\ (1)\\b=\frac{2}{a}\end{cases}$

Resolvendo a equação (1), vem que:

$$-3=a^2-\frac{4}{a^2}\Leftrightarrow -3a^2=a^4-4$$

$$\Leftrightarrow a^4+3a^2-4=0$$

Substituindo $a^2=y$, vem:

$$y^2+3y-4=0\Leftrightarrow y=-3\pm\frac{\sqrt{9+16}}{2}$$

$$\Leftrightarrow y=\frac{-3\pm5}{2}$$

$$\Leftrightarrow y=-4\vee y=1$$

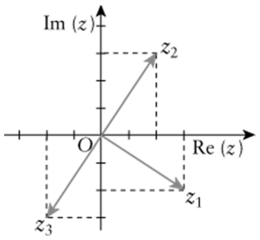
$y=-4$ é impossível, uma vez que $y=a^2$

$y=1\Leftrightarrow a^2=1\Leftrightarrow a=-1\vee a=1$

- Se $a=-1$ então $b=-2$
- Se $a=1$ então $b=2$

Logo, as raízes quadradas de – 3 + 4i são – 1 – 2i e 1 + 2i.

9.



$z_1 = 3 - 2i$

$z_2 = i\,(3 - 2i) = 3i - 2i^2 = 2 + 3i$

$z_3 = -i\,(3 - 2i) = -3i + 2i^2 = -2 - 3i$

a) $z_1 \curvearrowright z_2 \to R\,(0, 90°)$

b) $z_1 \curvearrowright z_3 \to R\,(0, -90°)$

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10. Mostrar que $\overline{z_1+z_2}=\overline{z_1}+\overline{z_2}$

Pretende-se mostrar que o conjugado da soma de dois complexos é igual à soma dos conjugados das parcelas.

Sejam $z_1 = a + bi$ e $z_2 = c + di$

1.º membro = $\overline{z_1+z_2}$

$$=\overline{(a+bi)+(c+di)}$$

$$=\overline{(a+c)+(b+d)i}$$

$$=(a+c)-(b+d)i$$

2.º membro = $\overline{z_1}+\overline{z_2}$

$$=\overline{(a+bi)}+\overline{(c+di)}$$

$$=(a-bi)+(c-di)$$

$$=(a+c)-(b+d)i$$

= 1.º membro

Logo, $\overline{z_1+z_2}=\overline{z_1}+\overline{z_2}$ c.q.m.

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11.1 **a)** $3x^2-2x+1=0$

$$\Leftrightarrow x=\frac{2\pm\sqrt{4-12}}{6}$$

$$\Leftrightarrow x=\frac{2\pm\sqrt{-8}}{6}\Leftrightarrow x=\frac{2\pm2i\sqrt{2}}{6}$$

$$\Leftrightarrow x=\frac{1}{3}-\frac{\sqrt{2}}{3}i\vee x=\frac{1}{3}+\frac{\sqrt{2}}{3}i$$

b) $ax^2+bx+c=0\Leftrightarrow$

$$\Leftrightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Como α e β são raízes da equação $ax^2+bx+c=0$

Então,

$$a(x-\alpha)(x-\beta)=ax^2+bx+c$$

$$a\left(x^2-\beta x-x\alpha+\alpha\beta\right)=ax^2+bx+c$$

$$ax^2-a(\alpha+\beta)x+a\alpha\beta=ax^2+bx+c$$

$$\begin{cases}a=a\\-a(\alpha+\beta)=b\\a\alpha\beta=c\end{cases}\Leftrightarrow$$

$$\begin{cases}\alpha+\beta=-\dfrac{b}{a}\\[2mm]\alpha\beta=\dfrac{c}{a}\end{cases}$$

11.2 Se – i é solução também 2 + i é solução.

Sendo S = soma das raízes e

P = Produto das raízes, daí que

$S = 2 - i + 2 + i = 4$

$P = (2 - i)\,(2 + i) = 4 - i^2 = 4 + 1 = 5$

Aplicando a fórmula $x^2 - S\,x + P = 0$, vem, por exemplo:

$x^2 - 4x + 5 = 0$

$2x^2 - 8x + 10 = 0$

$3x^2 - 12x + 15 = 0$

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12.1. $z_1 = 3$

módulo: $|z_1| = 3$

argumento: $\arg z_1 = 0$

12.2. $z_2 = 2 + 2\mathrm{i}$

módulo: $|z_2| = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$

$$\begin{cases} \operatorname{tg}(\arg z_1) = \dfrac{2}{2} = 1 \\ \arg z_1 \in 1.^{\circ} Q \end{cases} \Rightarrow \frac{\pi}{4} \text{ é argumento de } z_1$$

12.3. $z_3 = 5\mathrm{i}$

módulo: $|z_3| = 5$

argumento: $\arg z_3 = \dfrac{\pi}{2}$

12.4. $z_4 = -3 + \sqrt{3}\mathrm{i}$, a imagem geométrica de

$z_4 \curvearrowright \left(-3, \sqrt{3}\right) \in 2.^{\circ} Q$

módulo: $|z_4| = \sqrt{(-3)^2 + \left(\sqrt{3}^2\right)} = \sqrt{9+3}$

$= \sqrt{12} = 2\sqrt{3}$

$$\begin{cases} \operatorname{tg}(\arg z_4) = \dfrac{\sqrt{3}}{-3} \\ \arg z_4 \in 2.^{\circ} Q \end{cases} \Rightarrow \pi - \frac{\pi}{6} = \frac{5\pi}{6} \text{ é argumento de } z_4$$

12.5. $z_5 = -2 \in \mathbb{R}^-$

módulo: $|z_5| = 2$

argumento: $\arg z_5 = \pi$

12.6. $z_6 = -3 - 3\mathrm{i}$, a imagem geométrica de

$z_6 \curvearrowright (-3, -3) \in 3.^{\circ} Q$

módulo: $|z_6| = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = 3\sqrt{2}$

$$\begin{cases} \operatorname{tg}(\arg z_6) = \dfrac{-3}{-3} = 1 \\ \arg z_6 \in 3.^{\circ} Q \end{cases} \Rightarrow \pi + \frac{\pi}{4} = \frac{5\pi}{4} \text{ é argumento de } z_6$$

12.7. $z_7 = -\sqrt{2}\mathrm{i}$

módulo: $|z_7| = \left|-\sqrt{2}\mathrm{i}\right|\sqrt{2}$

argumento: $\arg z_7 = \dfrac{3\pi}{2}$

12.8. $z_8 = 1 - \sqrt{3}\mathrm{i}$

módulo: $|z_8| = \sqrt{1^2 + \left(-\sqrt{3}\right)^2} = \sqrt{1+3} = 2$

$$\begin{cases} \operatorname{tg}(\arg z_8) = \dfrac{-\sqrt{3}}{1} = -\sqrt{3} \\ \arg z_8 \in 4.^{\circ} Q \end{cases} \Rightarrow -\frac{\pi}{3} \text{ é argumento de } z_3$$

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13.1. $z_1 = 3 \in IR^+$

Logo, $\arg z_1 = 0$ e $|z_1| = 3$,

daí que $z_1 = 3 \operatorname{cis} 0$.

13.2. $z_2 = \sqrt{2} + \mathrm{i}$

A imagem geométrica de

$z_2 \curvearrowright \left(\sqrt{3}, 1\right) \in 1.^{\circ} Q$

$$\begin{cases} \operatorname{tg}(\arg z_2) = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3} \\ \arg z_1 \in 1.^{\circ} Q \end{cases} \Rightarrow \frac{\pi}{6} \text{ é argumento de } z_1$$

$|z_2| = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} = 2$

daí que $z_2 = 2 \operatorname{cis}\left(\dfrac{\pi}{6}\right)$.

13.3. $z_3 = 4\mathrm{i}$

$\arg z_3 = \dfrac{\pi}{2}$ e $|z_3| = 4$

daí que $z_3 = 4 \operatorname{cis}\left(\dfrac{\pi}{2}\right)$.

13.4. $z_4 = -4 - 4\mathrm{i}$

A imagem geométrica de

$z_4 \curvearrowright (-4, -4) \in 3.^{\circ} Q$

$$\begin{cases} \operatorname{tg}(\arg z_4) = \dfrac{-4}{-4} = 1 \\ \arg z_4 \in 3.^{\circ} Q \end{cases} \Rightarrow \pi + \frac{\pi}{4} = \frac{5\pi}{4} \text{ é argumento de } z_4$$

$|z_4| = \sqrt{|-4|^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

daí que $z_4 = 4\sqrt{2} \operatorname{cis} \dfrac{5\pi}{4}$.

13.5. $z_5 = -2\text{i}$

$\arg z_5 = \dfrac{3\pi}{2}$ e $|z_5| = 2$,

daí que $z_5 = 2\,\text{cis}\dfrac{3\pi}{2}$.

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14.1. $z = 2\left(\cos\dfrac{\pi}{3} + \text{i}\sin\dfrac{\pi}{3}\right)$

$= 2\left(\dfrac{1}{2} + \text{i}\dfrac{\sqrt{3}}{2}\right) = \dfrac{2}{2} + \dfrac{2\sqrt{3}}{2}\text{i} = 1 + \sqrt{3}\text{i}$

14.2. $5\,\text{cis}\left(-\dfrac{\pi}{4}\right) =$

$= 5\left(\cos\left(-\dfrac{\pi}{4}\right) + \text{i}\sin\left(-\dfrac{\pi}{4}\right)\right)$

$= 5\left(\dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{2}\text{i}\right)$

$= \dfrac{5\sqrt{2}}{2} - \dfrac{5\sqrt{2}}{2}\text{i}$

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15.1. **a)** $z \times \bar{z} = |z|^2$

Seja $z = \rho\text{cis}\theta$ então $\bar{z} = \rho\text{cis}(-\theta)$

Donde $z \times \bar{z} = (\rho\text{cis}\theta) \times (\rho\text{cis}(-\theta))$

$= \rho^2\text{cis}[\theta + (-\theta)]$

$= \rho^2\text{cis}0$

$= \rho^2[\cos 0 + \text{i}\sin 0]$

$= \rho^2[1 + \text{i} \times 0]$

$= \rho^2 = |z|^2$

Logo, $z \times \bar{z} = |z|^2$.

b) $\dfrac{1}{z} = \dfrac{\bar{z}}{|z|^2}$

Seja $z = \rho\text{cis}\theta$, então $\bar{z} = \rho\text{cis}(-\theta)$ e $|z| = \rho$

Temos que:

1.º membro $= \dfrac{1}{z} = \dfrac{1}{\rho\text{cis}(\theta)} = \dfrac{\text{cis}(\theta)}{\rho\text{cis}(\theta)\text{cis}(-\theta)}$

$= \dfrac{\text{cis}(\theta)}{\rho\text{cis}0} = \dfrac{\text{cis}(-\theta)}{\rho}$

2.º membro $= \dfrac{\bar{z}}{|z|^2} = \dfrac{\rho\text{cis}(-\theta)}{\rho^2}$

$= \dfrac{\text{cis}(-\theta)}{\rho}$ = 1.º membro

De outro modo:

Seja: $z = a + b\text{i}$, então $\bar{z} = a - b\text{i}$ e $|z| = \sqrt{a^2 + b^2}$

1.º membro $= \dfrac{1}{z} = \dfrac{1}{a + b\text{i}}$

$= \dfrac{a + b\text{i}}{(a + b\text{i})(a - b\text{i})} = \dfrac{a - b\text{i}}{a^2 - b^2\text{i}^2}$

$= \dfrac{a - b\text{i}}{a^2 + b^2}$

2.º membro $= \dfrac{\bar{z}}{|z|^2} = \dfrac{a - b\text{i}}{\left(\sqrt{a^2 + b^2}\right)^2}$

$= \dfrac{a - b\text{i}}{a^2 + b^2}$ = 1.º membro.

15.2. **a)** $z_1 \times z_2 = \left[\dfrac{1}{3}\text{cis}\left(\dfrac{\pi}{4}\right)\right] \times \left[5\text{cis}\left(-\dfrac{\pi}{4}\right)\right]$

$= \dfrac{1}{3} \times 5\text{cis}\left[\dfrac{\pi}{4} + \left(-\dfrac{\pi}{4}\right)\right] = \dfrac{5}{3}\text{cis}0 = \dfrac{5}{3}$

b) $z_1 \times z_3 = \left[\dfrac{1}{3}\text{cis}\left(\dfrac{\pi}{4}\right)\right] \times \left[3\text{cis}\left(\dfrac{3\pi}{4}\right)\right]$

$= \dfrac{1}{3} \times 3\text{cis}\left(\dfrac{\pi}{4} + \dfrac{3\pi}{4}\right)$

$= 1\text{cis}\left(\dfrac{4\pi}{4}\right) = \text{cis}\pi = -1$

c) $z_1 \times z_2 \times z_3 = (z_1 \times z_2) \times z_3$

$= \left[\dfrac{5}{3}\text{cis}(0)\right] \times \left[3\text{cis}\left(\dfrac{3\pi}{4}\right)\right]$ pela propriedade associativa da multiplicação

pela alínea **a)**

$= \dfrac{5}{3} \times 3\text{cis}\left(0 + \dfrac{3\pi}{4}\right) = 5\text{cis}\left(\dfrac{3\pi}{4}\right)$

$= 5\left(\cos\dfrac{3\pi}{4} + \text{i}\sin\dfrac{3\pi}{4}\right)$

$= 5\left(-\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}\text{i}\right)$

$= -\dfrac{5\sqrt{2}}{2} + \dfrac{5\sqrt{2}}{2}\text{i}$

15.3. **a)** $\bar{z} \times z_1 = \left(\sqrt{6}+\sqrt{2}i\right)\times\left(\sqrt{6}-\sqrt{2}i\right)$

$$= \left[2\sqrt{2}\text{cis}\left(\frac{\pi}{6}\right)\right]\times\left[2\sqrt{2}\text{cis}\left(-\frac{\pi}{6}\right)\right] \frac{5}{3}$$

$$= 8\text{cis}\left[\frac{\pi}{6}+\left(-\frac{\pi}{6}\right)\right] = 8\text{cis}0 = 8$$

Cálculo auxiliar

Seja $z_1 = \sqrt{6}-\sqrt{2}i$, então

$$|z_1| = \sqrt{\left(\sqrt{6}\right)^2+\left(-\sqrt{2}\right)^2} = \sqrt{6+2} = 2\sqrt{2}$$

$$\begin{cases} \text{tg}(\arg z_1) = \dfrac{-\sqrt{2}}{\sqrt{6}} = -\dfrac{\sqrt{3}}{3} \Rightarrow -\dfrac{\pi}{6} \text{ é argumento de } z_1 \\ \arg z_1 \in 4.^{\text{o}} Q \end{cases}$$

donde $z_1 = 2\sqrt{2}\,\text{cis}\left(-\frac{\pi}{6}\right)$ e $\bar{z}_1 = 2\sqrt{2}\,\text{cis}\left(\frac{\pi}{6}\right)$

b) $z_1 \times z_2 \times \bar{z}_3$

$$= 2\sqrt{2}\text{cis}\left(-\frac{\pi}{6}\right)\times 2\text{cis}\left(\frac{3\pi}{2}\right)\times 2\text{cis}(-\pi)$$

$$= \left(2\sqrt{2}\times 2\times 2\right)\text{cis}\left(-\frac{\pi}{6}+\frac{3\pi}{2}-\pi\right)$$

$$= 8\sqrt{2}\text{cis}\left(-\frac{\pi}{6}+\frac{9\pi}{2}-\frac{6\pi}{6}\right)$$

$$= 8\sqrt{2}\text{cis}\left(\frac{\pi}{3}\right)$$

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16.1. **a)** z^{29}, sendo $z = 1cis\pi$

$$z^{29} = (1\text{cis}\pi)^{29} = 1^{29}\text{cis}(29\pi) = 1\text{cis}(\pi) = \text{cis}\pi$$

b) $\left(-\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^{80}$

$$= \left[1cis\left(\frac{7\pi}{6}\right)\right]^{80}$$

$$= 1^{80}\text{cis}\left(\frac{7\pi}{6}\times 80\right)$$

$$= \text{cis}\left(\frac{560\pi}{6}\right)$$

$$= \text{cis}\left(\frac{4\pi}{3}\right)$$

$$= \text{cis}\left(-\frac{2\pi}{3}\right)$$

Cálculo auxiliar

Seja $z_1 = -\frac{\sqrt{3}}{2}-\frac{1}{2}i$

$$|z_1| = \sqrt{\left(-\frac{\sqrt{3}}{2}\right)^2+\left(-\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4}+\frac{1}{4}} = 1$$

$$\arg z_1 = \pi + \tan^{-1}\left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) = \pi + \tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$$

$= \pi + \frac{\pi}{6} = \frac{7\pi}{6}$ daí que $z_1 = \text{cis}\frac{7\pi}{6}$

c) $(1-i)^{-100}$

$$= \left[\sqrt{2}cis\left(-\frac{\pi}{4}\right)\right]^{-100}$$

$$= \left(\sqrt{2}\right)^{-100} cis\left(-\frac{\pi}{4}\times(-100)\right)$$

$$= \frac{1}{\left(\sqrt{2}\right)^{100}}cis(25\pi)$$

$$= \frac{1}{\left(2^{\frac{1}{2}}\right)^{100}}\text{ cis } \pi$$

$$= \frac{1}{2^{50}}\text{ cis } \pi$$

16.2. Seja $z = \text{cis}\theta$

a) $z+\bar{z} = 2\cos\theta$

Se $z = \text{cis}\theta$ então $\bar{z} = \text{cis}(-\theta)$, sendo:

1.º membro $= z+\bar{z} = \text{cis}\theta + \text{cis}(-\theta)$

$= (\cos\theta + i\sin\theta) + (\cos(-\theta) + i\sin(-\theta))$

$= \cos\theta + i\sin\theta + \cos\theta - i\sin\theta$

$= 2\cos\theta$ = 2.º membro

b) $z^2+\bar{z}^2 = 2\cos(2\theta)$

1.º membro $= z^2+\bar{z}^2 = (\text{cis}\theta)^2 + (\text{cis}(-\theta))^2$

$= \text{cis}(2\theta) + \text{cis}(-2\theta)$

$= \cos(2\theta) + i\sin(2\theta) + \cos(-2\theta) + i\sin\theta(-2\theta)$

$= \cos(2\theta) + i\sin(2\theta) + \cos(2\theta) - i\sin(2\theta)$

$= 2\cos(2\theta)$ = 2.º membro

c) $z^n + \bar{z}^n = 2\cos(n\theta)$

1.º membro = $z^n + \bar{z}^n = (\text{cis}\,\theta)^n + (\text{cis}\,(-\theta))^n$

$= \text{cis}(n\theta) + \text{cis}(-n\theta)$

$= \cos(n\theta) + i\sin(n\theta) + \cos(-n\theta) + i\sin\theta(-n\theta)$

$= \cos(n\theta) + i\sin(n\theta) + \cos(n\theta) - i\sin(n\theta)$

$= 2\cos(n\theta) =$ 2.º membro

d) $\sin(n\theta) = \dfrac{z^n - \bar{z}^n}{2i}$

2.º membro = $\dfrac{z^n - \bar{z}^n}{2i}$

$= \dfrac{(\text{cis}2\theta)^n - (\text{cis}(-\theta)^n)}{2i}$

$= \dfrac{\text{cis}(n\theta) - \text{cis}(-n\theta)}{2i}$

$= \dfrac{\cos(n\theta) + i\sin(n\theta) - (\cos(-n\theta) + i\sin(-n\theta))}{2i}$

$= \dfrac{\cos(n\theta) + i\sin(n\theta) + \cos(n\theta) + i\sin(n\theta)}{2i}$

$= \dfrac{2i\sin(n\theta)}{2i}$

$= \sin(n\theta) =$ 1.º membro

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17.1. $\dfrac{2i}{\text{cis}\left(\frac{\pi}{6}\right)} = \dfrac{2\text{cis}\frac{\pi}{2}}{\text{cis}\left(\frac{\pi}{6}\right)}$

$= 2\text{cis}\left(\frac{\pi}{2} - \frac{\pi}{6}\right) = 2\text{cis}\left(\frac{3\pi}{6} - \frac{\pi}{6}\right) = 2\text{cis}\left(\frac{\pi}{3}\right)$

17.2. $\left(\dfrac{\frac{\sqrt{3}}{2} + \frac{1}{2}i}{1+i}\right)^2 = \left[\dfrac{\text{cis}\left(\frac{\pi}{6}\right)}{\sqrt{2}\text{cis}\left(\frac{\pi}{4}\right)}\right]^2$

$= \left[\frac{1}{\sqrt{2}}\text{cis}\left(\frac{\pi}{6} - \frac{\pi}{4}\right)\right]^2$

$= \left[\frac{\sqrt{2}}{2}\text{cis}\left(\frac{2\pi}{12} - \frac{3\pi}{12}\right)\right]^2$

$= \left[\frac{\sqrt{2}}{2}\text{cis}\left(-\frac{\pi}{12}\right)\right]^2$

$= \left(\frac{\sqrt{2}}{2}\right)^2 \text{cis}\left(-\frac{\pi}{12} \times 2\right) = \frac{1}{2}\text{cis}\left(-\frac{\pi}{6}\right)$

Cálculo auxiliar

Seja $z_1 = \frac{\sqrt{3}}{2} + \frac{1}{2}i$

$|z_1| = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4} + \frac{1}{4}} = 1$

$\arg z_1 = \text{tg}^{-1}\left(\dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) = \text{tg}^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$ é argumento de z_1

$z_1 = \text{cis}\left(\frac{\pi}{6}\right)$

Seja $z_2 = 1 + i$

$|z_2| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\begin{cases} \text{tg}(\arg z_2) = 1 \\ \arg z_2 \in 1.^{\text{o}}Q \end{cases} \Rightarrow \frac{\pi}{4}$ é argumento de z_2

$z_2 = \sqrt{2}\,\text{cis}\left(\frac{\pi}{4}\right)$

17.3. $\left(\dfrac{-1 + i^{15}}{\sqrt{3} + i}\right)^5 = \left(\dfrac{-1 + (i^4)^3 \times i^3}{\sqrt{3} + i}\right)^5$

$= \left(\dfrac{-1 + i}{\sqrt{3} + i}\right)^5 = \left[\dfrac{\sqrt{2}\text{cis}\left(\frac{5\pi}{4}\right)}{2\text{cis}\left(\frac{\pi}{6}\right)}\right]^5$

$= \left[\frac{\sqrt{2}}{2}\text{cis}\left(\frac{5\pi}{4} - \frac{\pi}{6}\right)\right]^5$

$= \left(\frac{\sqrt{2}}{2}\right)^5 \text{cis}\left(5\left(\frac{15\pi}{12} - \frac{2\pi}{12}\right)\right)$

$= \frac{4\sqrt{2}}{32}\text{cis}\left[5\left(\frac{13\pi}{12}\right)\right] = \frac{\sqrt{2}}{8}\text{cis}\left(\frac{65\pi}{12}\right)$

$= \frac{\sqrt{2}}{8}\text{cis}\left(\frac{17\pi}{12}\right)$

Cálculo auxiliar

Seja $z_1 = -1-i$

$|z_1| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$

$\begin{cases} tg(\arg z_1) = \frac{-1}{-1} = 1 \\ \arg z_1 \in 3.^{o}Q \end{cases} \Rightarrow \pi + \frac{\pi}{4} = \frac{5\pi}{4}$ é argumento de z_1

$z_1 = \text{cis}\left(\frac{\pi}{6}\right)$

Seja $z_2 = \sqrt{3} + i$

$|z_2| = \sqrt{(\sqrt{3})^2 + (+1)^2} = \sqrt{4} = 2$

$\begin{cases} tg(\arg z_2) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \\ \arg z_2 \in 1.^{o}Q \end{cases} \Rightarrow \frac{\pi}{6}$ é argumento de z_2

logo $z_2 = 2\,\text{cis}\left(\frac{\pi}{6}\right)$

17.4. $\frac{i^{513} - (1+i)^4}{1+i} - i = \frac{(i)^4 \times i - (1+i)^4}{1+i}$

$= \frac{i - (1+i)^4}{1+i} - i = \frac{i+4}{1+i} - i$

$= \frac{i+4}{1+i} - \frac{i(1+i)}{(1+i)}$

$= \frac{i+4-i-i^2}{1+i} = -\frac{5}{1+i} = \frac{5\text{cis}0}{\sqrt{2}\text{cis}\left(\frac{\pi}{4}\right)}$

Cálculo auxiliar

Seja $w = 1+i$

$|w| = \sqrt{2}$

$\begin{cases} tg(\arg w) = \frac{1}{1} = 1 \\ \arg w \in 1.^{o}Q \end{cases} \Rightarrow \frac{\pi}{4}$ é argumento de z

$w = \sqrt{2}\,\text{cis}\left(\frac{\pi}{4}\right)$

$(1+i)^4 = w^4 = (\sqrt{2})^4 \text{cis}\left(4 \times \frac{\pi}{4}\right) = 4\text{cis}\pi$

$= 4(\cos\pi + i\sin\pi)$

$= 4(-1 + i \times 0) = -4$

17.5. $\left(\frac{3-4i-\frac{1}{i}}{-3i^{22}}\right)^5 = \left(\frac{3-4i-\frac{-i}{i(-i)}}{-3i^{22}}\right)^5$

$= \left(\frac{3-4i-\frac{-i}{-i^2}}{-3(i^4)^5 i^2}\right)^5 = \left(\frac{3-4i+i}{-3(i^2)}\right)^5$

$= \left(\frac{3-3i}{3}\right)^5 = (1-i)^5 = \left(\sqrt{2}\,\text{cos}\left(-\frac{\pi}{4}\right)\right)^5$

$= 4\sqrt{2}\text{cis}\left(-\frac{5\pi}{4}\right) = 4\sqrt{2}\text{cis}\left(-\frac{5\pi}{4} + 2\pi\right)$

$= 4\sqrt{2}\,\text{cis}\left(\frac{3\pi}{4}\right)$

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18.1. **a)** $\sqrt{(\sqrt{3}-i)} = \sqrt{2\text{cis}\left(-\frac{\pi}{6}\right)}$

$= \sqrt{2}\text{cis}\frac{-\frac{\pi}{6} + 2k\pi}{2}, k \in \{0,1\}$

Se $k = 0$, $z_0 = \sqrt{2}\,\text{cis}\left(-\frac{\pi}{12}\right)$

Se $k = 1$, $z_1 = \sqrt{2}\,\text{cis}\left(\frac{11\pi}{12}\right)$

b) $\sqrt[4]{\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)} = \sqrt[4]{1\text{cis}\left(\frac{3\pi}{4}\right)}$

$= \sqrt[4]{1}\text{cis}\frac{\frac{3\pi}{4} + 2k\pi}{4}, k \in \{0,1,2,3\}$

$= \text{cis}\frac{3\pi + 8k\pi}{16}, k \in \{0,1,2,3\}$

Se $k = 0 \rightarrow z_0 = \text{cis}\left(\frac{3\pi}{16}\right)$

Se $k = 1 \rightarrow z_1 = \text{cis}\left(\frac{11\pi}{16}\right)$

Se $k = 2 \rightarrow z_2 = \text{cis}\left(\frac{19\pi}{16}\right)$

Se $k = 3 \rightarrow z_3 = \text{cis}\left(\frac{27\pi}{16}\right)$

Cálculo auxiliar

Seja $w=\sqrt{3}-\mathrm{i}$

$|w|=\sqrt{\left(\sqrt{3}\right)^2+(-1)^2}=2$

$\begin{cases} tg(\arg w)=\dfrac{-1}{\sqrt{3}}=-\dfrac{\sqrt{3}}{3} \\ \arg w\in 4.^{\text{o}}Q \end{cases} \Rightarrow -\dfrac{\pi}{6}$ é argumento de w

Seja $z=-\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{2}}\mathrm{i}=-\dfrac{\sqrt{2}}{2}+\dfrac{\sqrt{2}}{2}\mathrm{i}$

$|z|=\sqrt{\left(-\dfrac{\sqrt{2}}{2}\right)^2+\left(\dfrac{\sqrt{2}}{2}\right)^2}=\sqrt{\dfrac{2}{4}+\dfrac{2}{4}}=1$

$\begin{cases} tg(\arg z)=\dfrac{\sqrt{2}}{-\sqrt{2}}=-1 \\ \arg z\in 2.^{\text{o}}Q \end{cases} \Rightarrow \pi-\dfrac{\pi}{4}=\dfrac{3\pi}{4}$ é argumento de z

logo $z=1\operatorname{cis}\left(\dfrac{3\pi}{4}\right)$

c) $\sqrt[5]{-32\mathrm{i}}=\sqrt[5]{32\operatorname{cis}\left(\dfrac{3\pi}{2}\right)}$

$=\sqrt[5]{32}\operatorname{cis}\dfrac{\dfrac{3\pi}{2}+2k\pi}{5}, k\in\{0,1,2,3,4\}$

$=2\operatorname{cis}\dfrac{\dfrac{3\pi}{2}+2k\pi}{5}, k\in\{0,1,2,3,4\}$

$=2\operatorname{cis}\dfrac{3\pi+4k\pi}{10}, k\in\{0,1,2,3,4\}$

Se $k=0\rightarrow z_0=2\operatorname{cis}\left(\dfrac{3\pi}{10}\right)$

Se $k=1\rightarrow z_1=2\operatorname{cis}\left(\dfrac{7\pi}{10}\right)$

Se $k=2\rightarrow z_2=2\operatorname{cis}\left(\dfrac{11\pi}{10}\right)$

Se $k=3\rightarrow z_3=2\operatorname{cis}\left(\dfrac{3\pi}{10}\right)$

Se $k=4\rightarrow z_4=2\operatorname{cis}\left(\dfrac{19\pi}{10}\right)$

18.2. a) $z^3-1=0\Leftrightarrow z^3=1\Leftrightarrow z=\sqrt[3]{1}$

$\Leftrightarrow z=\sqrt[3]{1\operatorname{cis}0}\Leftrightarrow z=\sqrt[3]{1}\operatorname{cis}\dfrac{0+2k\pi}{3}, k\in\{0,1,2\}$

$\Leftrightarrow z=\operatorname{cis}\left(\dfrac{2k\pi}{3}\right), k\in\{0,1,2\}$

$k=0\rightarrow z_0=\operatorname{cis}0$

$k=1\rightarrow z_1=\operatorname{cis}\dfrac{2\pi}{3}$

$k=2\rightarrow z_2=\operatorname{cis}\dfrac{4\pi}{3}$, logo

$S=\left\{\operatorname{cis}0, \operatorname{cis}\dfrac{2\pi}{3}, \operatorname{cis}\dfrac{4\pi}{3}\right\}$

b) $z^4+1=0\Leftrightarrow z^4=-1\Leftrightarrow z=\sqrt[4]{-1}$

$\Leftrightarrow z=\sqrt[4]{1\operatorname{cis}\pi}\Leftrightarrow z=\sqrt[4]{1}\operatorname{cis}\dfrac{\pi+2k\pi}{4}, k\in\{0,1,2,3\}$

$\Leftrightarrow z=\operatorname{cis}\dfrac{\pi+2k\pi}{4}, k\in\{0,1,2,3\}$

$k=0\rightarrow z_0=\operatorname{cis}\dfrac{\pi}{4}$

$k=1\rightarrow z_1=\operatorname{cis}\dfrac{3\pi}{4}$

$k=2\rightarrow z_2=\operatorname{cis}\dfrac{5\pi}{4}$

$k=3\rightarrow z_3=\operatorname{cis}\dfrac{7\pi}{4}$, logo

$S=\left\{\operatorname{cis}\dfrac{\pi}{4}, \operatorname{cis}\dfrac{3\pi}{4}, \operatorname{cis}\dfrac{5\pi}{4}, \operatorname{cis}\dfrac{7\pi}{4}\right\}$

c) $z^6+\mathrm{i}-1=0\Leftrightarrow z^6=1-\mathrm{i}$

$\Leftrightarrow z=\sqrt[6]{1-\mathrm{i}}$

Como $1-\mathrm{i}=\sqrt{2}\operatorname{cis}\left(-\dfrac{\pi}{4}\right)$, vem:

$z=\sqrt[6]{\sqrt{2}\operatorname{cis}\left(-\dfrac{\pi}{4}\right)}\Leftrightarrow$

$\Leftrightarrow z=\sqrt[12]{2}\operatorname{cis}\dfrac{-\dfrac{\pi}{4}+2k\pi}{6}, k\in\{0,1,2,3,4,5\}$

$\Leftrightarrow z=\sqrt[12]{2}\operatorname{cis}\dfrac{-\pi+8k\pi}{24}, k\in\{0,1,2,3,4,5\}$

$k=0\rightarrow z_0=\sqrt[12]{2}\operatorname{cis}\left(-\dfrac{\pi}{24}\right)$

$k=1 \to z_1 = \sqrt[12]{2}\text{cis}\left(\frac{7\pi}{24}\right)$

$k=2 \to z_2 = \sqrt[12]{2}\text{cis}\left(\frac{15\pi}{24}\right) = \sqrt[12]{2}\text{cis}\left(\frac{5\pi}{8}\right)$

$k=3 \to z_3 = \sqrt[12]{2}\text{cis}\left(\frac{23\pi}{24}\right)$

$k=4 \to z_4 = \sqrt[12]{2}\text{cis}\left(\frac{31\pi}{24}\right)$

$k=5 \to z_5 = \sqrt[12]{2}\text{cis}\left(\frac{39\pi}{24}\right) = \sqrt[12]{2}\text{cis}\left(\frac{13\pi}{8}\right)$

logo:

$$S = \left\{\sqrt[12]{2}\text{cis}\left(-\frac{\pi}{24}\right), \sqrt[12]{2}\text{cis}\left(\frac{7\pi}{12}\right),\right.$$
$$\sqrt[12]{2}\text{cis}\left(\frac{5\pi}{8}\right), \sqrt[12]{2}\text{cis}\left(\frac{23\pi}{24}\right)$$
$$\left.\sqrt[12]{2}\text{cis}\left(\frac{31\pi}{24}\right), \sqrt[12]{2}\text{cis}\left(\frac{13\pi}{8}\right)\right\}$$

d) $z^3 = \bar{\bar{z}} \Leftrightarrow z^3 = z$

$\Leftrightarrow z^3 - z = 0$

$\Leftrightarrow z(z^2 - 1) = 0$

$\Leftrightarrow z = 0 \vee z = \sqrt{1}$

$\Leftrightarrow z = 0 \vee z = \sqrt{1\text{cis}0}$

$\Leftrightarrow z = 0 \vee z = \sqrt{1}\text{cis}\frac{0+2k\pi}{2}, k \in \{0,1\}$

$\Leftrightarrow z = 0 \vee z = \text{cis}(0) \vee z = \text{cis}\pi$

$\Leftrightarrow z = 0 \vee z = 1 \vee z = -1$

$S = \{-1, 0, 1\}$

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19. $-64 = 64 \text{ cis } \pi$

$\sqrt[6]{-64} = \sqrt[6]{-64}\text{cis}\frac{\pi + 2k\pi}{6}$

$k \in \{0,1,2,3,4,5\}$

Então:

$z_0 = 2\text{cis}\left(\frac{\pi}{6}\right)$

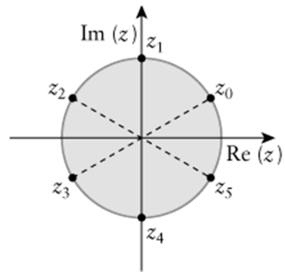
$z_1 = 2\text{cis}\left(\frac{3\pi}{6}\right) = 2\text{cis}\left(\frac{\pi}{2}\right)$

$z_2 = 2\text{cis}\left(\frac{5\pi}{6}\right)$

$z_3 = 2\text{cis}\left(\frac{7\pi}{6}\right)$

$z_4 = 2\text{cis}\left(\frac{9\pi}{6}\right) = 2\text{cis}\left(\frac{3\pi}{2}\right)$

$z_5 = 2\text{cis}\left(\frac{11\pi}{6}\right)$



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20.1. Sabemos que $z_1 = 1 + \sqrt{3}i$,

daí que:

$|z_1| = \sqrt{1^2 + \left(\sqrt{3}\right)^2} = 2$

$\arg(z_1) = tg^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$

Então: $z_1 = 2\text{cis}\left(\frac{\pi}{3}\right)$; assim, vem:

$z_2 = 2\text{cis}\left(\frac{\pi}{3} + \frac{2\pi}{3}\right) = 2\text{cis}(\pi) = -2$

$z_3 = 2\text{cis}\left(\pi + \frac{2\pi}{3}\right) = 2\text{cis}\left(\frac{5\pi}{3}\right)$

$= 2\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 1 - \sqrt{3}i$

20.2. Calculando a soma, na forma algébrica $z_1 + w$, $z_2 + w$ e $z_3 + w$, obtém-se o transformado do triângulo de vértices z_1, z_2 e z_3 pela transformação $T_{\vec{u}}$, assim:

$z'_1 = z_1 + w = 1 + \sqrt{3}i + (-i) = 1 + \left(\sqrt{3} - 1\right)i$

$z'_2 = z_2 + w = 1 - \sqrt{3}i + (-i) = 1 + \left(-\sqrt{3} - 1\right)i$

$z'_3 = z_3 + w = 1 - \sqrt{3}i + (-i) = 1 + \left(-\sqrt{3} - 1\right)i$

20.3. $z_1 \times z = 2\,\mathrm{cis}\left(\frac{\pi}{3}\right) \times \mathrm{cis}\left(\frac{\pi}{3}\right)$

$= 2\,\mathrm{cis}\left(\frac{\pi}{3} + \frac{\pi}{3}\right)$

$= 2\,\mathrm{cis}\left(\frac{2\pi}{3}\right)$

$z_2 \times z = 2\,\mathrm{cis}(\pi) \times \mathrm{cis}\left(\frac{\pi}{3}\right)$

$= 2\,\mathrm{cis}\left(\pi + \frac{\pi}{3}\right)$

$= 2\,\mathrm{cis}\left(\frac{4\pi}{3}\right)$

$z_3 \times z = 2\,\mathrm{cis}\left(\frac{5\pi}{3}\right) \times \mathrm{cis}\left(\frac{\pi}{3}\right)$

$= 2\,\mathrm{cis}\left(\frac{5\pi}{3} + \frac{\pi}{3}\right)$

$= 2\,\mathrm{cis}(2\pi)$

Interpretação: O produto de $\mathrm{cis}\left(\frac{\pi}{3}\right)$ por cada número complexo correspondente a cada vértice do triângulo significa que se obtém uma rotação de centro (0, 0) e ângulo igual a $\frac{\pi}{3}$, de cada um dos vértices do triângulo.

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1. Seja $z = r\mathrm{cis}\theta$

então o seu simétrico é $-z = r\mathrm{cis}(\pi + \theta)$

Logo, a resposta correcta é a (B), uma vez que:

$\mathrm{cis}(\alpha - \pi) = \mathrm{cis}(\alpha + 2\pi - \pi) = \mathrm{cis}(\alpha + \pi)$

2. $z = 3\mathrm{cis}\left(\theta - \frac{\pi}{3}\right)$

Para que z seja imaginário puro de coeficiente positivo, temos:

$\theta - \frac{\pi}{3} = \frac{\pi}{2} + 2k\pi, k \in IZ$

$\Leftrightarrow \theta = \frac{5\pi}{6} + 2k\pi, k \in IZ$

Por exemplo, $\theta = \frac{5\pi}{6}$

Logo, a resposta correcta é a **(B).**

3. $z = i^{20} + i$

$z = (i^4)^5 + i$

$z = 1 + i$

módulo: $|z| = \sqrt{2}$

argumento: $\arg z = \frac{\pi}{4}$, então:

$z = \sqrt{2}\mathrm{cis}\left(\frac{\pi}{4}\right)$, a resposta correcta é a **(C).**

4. As imagens geométricas de números conjugados são simétricas relativamente ao eixo real.

Donde a resposta correcta é a (D).

5. $2w\mathrm{cis}(-\pi) = -2w$, representa o dobro do simétrico de w.

A resposta correcta é a (C).

6. i – w = i – (2 + 1) =i – 2 – i = – 2

Donde i – w é um real, logo, a resposta correcta será a (A).

7. Sendo $z = 2\mathrm{cis}\left(\frac{\pi}{9}\right)$,

então, $\frac{1}{z^2} = \frac{1}{4\mathrm{cis}\left(\frac{2\pi}{9}\right)} = \frac{1\mathrm{cis}0}{4\mathrm{cis}\left(\frac{2\pi}{9}\right)}$

$= \frac{1}{4}\mathrm{cis}\left(-\frac{2\pi}{9}\right)$

Logo, a resposta correcta é a **(C)**.

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1.1. $(z-3)^2 (1-2i)^2 - i^9$

$= [(3-i)-3]^2 (1-2i)^2 - (i^4)^2 \times i$

$= (-i)^2 (1 - 4i + 4i^2) - i$

$= -(1 - 4i - 4) - i$

$= 3 + 4i - i$

$= 3 + 3i$

1.2. Inverso do conjugado de w é $\frac{1}{\bar{w}}$, isto é, $\frac{1}{3-3i}$, donde:

$\frac{1}{3-3i} = \frac{3+3i}{(3-3i)(3+3i)} = \frac{3+3i}{9+(3i)^2}$

$= \frac{3+3i}{9-9i^2} = \frac{3+3i}{18} = \frac{3}{18} + \frac{3}{18}i = \frac{1}{6} + \frac{1}{6}i$

Na forma trigonométrica:

$$\left|\frac{1}{w}\right| = \sqrt{\left(\frac{1}{6}\right)^2 + \left(\frac{1}{6}\right)^2} = \sqrt{\frac{1}{36} + \frac{1}{36}} = \frac{\sqrt{2}}{6}$$

$$\begin{cases} tg\left(\arg\left(\dfrac{1}{w}\right)\right) = 1 \\ \arg\left(\dfrac{1}{w}\right) \in 1.^{\text{o}}Q \end{cases} \Rightarrow \frac{\pi}{4} \text{ é um argumento de } \frac{1}{w}$$

Daí que: $\dfrac{1}{w} = \dfrac{\sqrt{2}}{6}\text{cis}\left(\dfrac{\pi}{4}\right)$.

2.1. O polinómio $x^3 - 3x^2 + x + 5$ é divisível por $x + 1$, porque $P(-1) = 0$.

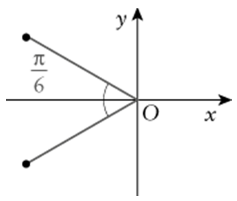
2.2. $x^3 - 3x^2 + x + 5 = (x+1)(x^2 - 4x + 5)$

$$x^2 - 4x + 5 = 0 \Leftrightarrow x = \frac{4 \pm \sqrt{16-20}}{2} \Leftrightarrow$$

$$\Leftrightarrow x = 2 + \text{i} \vee x = 2 - \text{i}$$

Os zeros do polinómio são -1, $2 + \text{i}$ e $2 - \text{i}$.

3. Do enunciado podemos retirar que:

- $|z| = |\bar{z}| = 1$
- z é da forma $z = \text{cis}\theta$ com $\dfrac{\pi}{2} < \theta < \pi$.
- 

As imagens de z e $\bar{z}$ são simétricas em relação ao eixo real.

Donde: $\arg(z) = \pi - \dfrac{\pi}{6} = \dfrac{5\pi}{6} \Rightarrow \arg(\bar{z}) = -\dfrac{5\pi}{6}$

3.1. $\text{i}z = \text{cis}\dfrac{\pi}{2}\text{cis}\left(\dfrac{5\pi}{6}\right) = \text{cis}\left(\dfrac{\pi}{2} + \dfrac{5\pi}{6}\right)$

$$= \text{cis}\left(\frac{4\pi}{3}\right) = -\frac{1}{2} - \frac{\sqrt{3}}{2}\text{i}$$

3.2. $\dfrac{\bar{z}}{\text{i}} = \dfrac{\text{cis}\left(-\dfrac{5\pi}{6}\right)}{\text{cis}\left(\dfrac{\pi}{2}\right)} = \text{cis}\left(-\dfrac{4\pi}{6} - \dfrac{\pi}{2}\right)$

$$= \text{cis}\left(-\frac{4\pi}{3}\right) = -\frac{1}{2} + \frac{\sqrt{3}}{2}\text{i}$$

4. $z_1^3 = z_2^2 \times w$

$$\Leftrightarrow (2+\text{i}) = \left(\sqrt{3}\text{cis}\left(\frac{\pi}{3}\right)\right)^2 w$$

$$\Leftrightarrow (2+\text{i})^2(2+\text{i}) = 3\text{cis}\frac{2\pi}{3}w$$

$$\Leftrightarrow (4+4\text{i}+\text{i}^2)(2+\text{i}) = 3\text{cis}\frac{2\pi}{3}w$$

$$\Leftrightarrow (3+4\text{i})(2+\text{i}) = 3\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}\text{i}\right)w$$

$$\Leftrightarrow (6+3\text{i}+8\text{i}+4\text{i}^2) = -\frac{3}{2} + \frac{3\sqrt{3}}{2}\text{i}$$

$$\Leftrightarrow (2+11\text{i}) = -\frac{3}{2} + \frac{3\sqrt{3}}{2}\text{i}$$

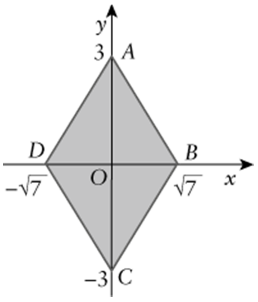
$$\Leftrightarrow \frac{2+11\text{i}}{-\frac{3}{2} + \frac{3\sqrt{3}}{2}\text{i}} = w$$

$$\Leftrightarrow \frac{(2+11\text{i})\left(-\frac{3}{2} + \frac{3\sqrt{3}}{2}\text{i}\right)}{\frac{9}{4} + \frac{27}{4}}$$

$$\Leftrightarrow w = \frac{-3 - 3\sqrt{3}i - \frac{32}{2}\text{i} + \frac{33\sqrt{3}}{2}}{9}$$

$$\Leftrightarrow w = \frac{11\sqrt{3} - 2}{6} - \frac{2\sqrt{3} + 11}{6}\text{i}$$

5. $z_1 = 3i$, imediatamente $z_2 = -3i$ é também um número complexo cuja imagem geométrica é um vértice do losango.



Perímetro do losango é 16, donde $\overline{AB} = 4$, logo

$$\overline{AB}^2 = \overline{OB}^2 + \overline{OA}$$

$$16 = 9 + \overline{OB}^2 \Leftrightarrow 16 - 9 = \overline{OB}^2 \Leftrightarrow 7 = \overline{OB}^2$$

$$\overline{OB} = \sqrt{17}$$

Resposta: $-3i$, $\sqrt{7}$ e $-\sqrt{7}$

6. $$\left(\sqrt{2}\operatorname{cis}\frac{\pi}{4}\right)z = 1 + i$$

$$\Leftrightarrow \left[\left(\sqrt{2}\right)^3 \operatorname{cis}\left(\frac{2\pi}{4}\right)\right]z = \sqrt{2}\operatorname{cis}\frac{\pi}{4}$$

$$\Leftrightarrow z = \frac{\sqrt{2}\operatorname{cis}\frac{\pi}{4}}{\left(\sqrt{2}\right)^3 \operatorname{cis}\frac{3\pi}{4}}$$

$$\Leftrightarrow z = \frac{\operatorname{cis}\frac{\pi}{4}}{2\operatorname{cis}\frac{3\pi}{4}} \Leftrightarrow z = \frac{1}{2}\operatorname{cis}\left(-\frac{\pi}{2}\right)$$

7. Do enunciado, sabemos:

- $z_1 = \overline{OA}\operatorname{cis}\left(\frac{\pi}{4}\right)$
- $z_2 = 6\sqrt{2}\operatorname{cis}(\theta_2)$
- a área do retângulo [$OABC$] é 12.

Assim, vem:

$A_{[OABC]} = \overline{OA} \times \overline{OC}$, mas $\overline{OC} = 6\sqrt{2}$, donde:

$$12 \times \overline{OA} \times 6\sqrt{2} \Leftrightarrow \overline{OA} = \frac{12}{6\sqrt{2}}$$

$$\Leftrightarrow \overline{OA} = \frac{2}{\sqrt{2}} \Leftrightarrow \overline{OA} = \frac{2\sqrt{2}}{2} \Leftrightarrow \overline{OA} = \sqrt{2}$$

Assim: $z_1 = \sqrt{2}\operatorname{cis}\left(\frac{\pi}{4}\right)$

$$= \sqrt{2}\left[\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right]$$

$$= \sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = 1 + i$$

Como $\arg(z_1) = \frac{\pi}{4}$ e $A\hat{O}C = \frac{\pi}{2}$, então:

$\arg(z_2) = \frac{\pi}{4} + \frac{\pi}{2}$, logo $\arg(z_2) = \frac{3\pi}{4}$

$$z_2 = 6\sqrt{2}\operatorname{cis}\left(\frac{3\pi}{4}\right) = 6\sqrt{2}\left[\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right]$$

$$= 6\sqrt{2}\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = -6 + 6i$$

Resposta: $z_1 = 1 + i$ e $z_2 = -6 + 6i$

8. z_1 tem um argumento $\frac{\pi}{3}$, logo $z_1 = r_1\operatorname{cis}\frac{\pi}{3}$

z_2 tem módulo $3\sqrt{3}$, logo $z_2 = 3\sqrt{3}\operatorname{cis}\theta_2$

Então: $z_2 \times \overline{z_2} + \left(\frac{z_1}{|z_1|}\right)^{10}$

$$= 3\sqrt{3}\operatorname{cis}(\theta_2) \times 3\sqrt{3}(-\theta_2) + \left(\frac{r_1\operatorname{cis}\frac{\pi}{3}}{r_1}\right)^{10}$$

$$= \left(3\sqrt{3}\right)^2 \operatorname{cis}(0) + \left(\operatorname{cis}\frac{\pi}{3}\right)^{10}$$

$$= 27\operatorname{cis}(0) + 1^{10}\operatorname{cis}\frac{10\pi}{3}$$

$$= 27(\cos 0 + i\sin 0) + \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$$

$$= 27(1 + i \times 0) + \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$$

$$= 27 - \frac{1}{2} - \frac{\sqrt{3}}{2}i = \frac{53}{2} - \frac{\sqrt{3}}{2}i$$

9. A imagem geométrica de z pertence ao 4.° Quadrante (eixos não incluídos), logo z será da forma:

$z = r\text{cis}\theta$, com $-\frac{\pi}{2} < \theta < 0$

$z^3 = r^3\text{cis}(3\theta)$

E como: $-\frac{\pi}{2} < \theta < 0$

Então: $-\frac{3\pi}{2} < 3\theta < 0$.

Daí que a imagem geométrica de $z^3 = r^3\text{cis}(3\theta)$,

com $-\frac{3\pi}{2} < 3\theta < 0$, não pode pertencer ao 1.° quadrante, mas pode pertencer ao 2.°, 3.° ou 4.° quadrante.

10. Seja $z = r\text{cis}\theta$, tal que, $\theta = \frac{3\pi}{4}$ ou $\theta = -\frac{\pi}{4}$

Uma vez que a imagem geométrica de z pertence à bissetriz dos quadrantes pares.

Assim:

$z^{20} = r^{20}\text{cis}\left(\frac{60\pi}{4}\right)$ ou $z^{20} = r^{20}\text{cis}\left(-\frac{20\pi}{4}\right)$

ou seja:

$z^{20} = r^{20}\text{cis}(\pi)$ ou $z = r^{20}\text{cis}(-\pi)$

Logo, a imagem geométrica de z^{20} pertence ao eixo real, uma vez que $\arg(z^{20}) = \pi$

Então, uma equação da recta à qual pertence a imagem geométrica de z^{20} é $y = 0$ ou $\text{Im}(z) = 0$.