PROPOSTAS DE RESOLUÇÃO

Capítulo 7

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1. $f(x)=\frac{1}{\sqrt{x}}$

$F(x)=2\sqrt{x}+C$

Como $F(4)=4$ então $2\sqrt{x}+C=4 \Leftrightarrow C=0$ logo a primitiva pedida é:

$F(x)=2\sqrt{x}$.

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2.1 $\int -7dx=-7x+C$

2.2 $\int -\frac{2}{3}dx=-\frac{2}{3}x+C$

2.3 $\int \sqrt{2}dx=\sqrt{2}x+C$

2.4 $\int -7xdx=-7\int xdx=-7\frac{x^2}{2}+C=-\frac{7}{2}x^2+C$

2.5 $\int \frac{7}{5}xdx=\frac{7}{5}\int xdx=\frac{7}{5}\frac{x^2}{2}+C=\frac{7}{10}x^2+C$

2.6 $\int x^3dx=\frac{x^{3+1}}{3+1}+C=\frac{x^4}{4}+C$

2.7 $\int x^{80}dx=\frac{x^{80+1}}{80+1}+C=\frac{x^{81}}{81}+C$

2.8 $\int \sqrt[3]{x}dx=\int x^{\frac{1}{3}}Cx=\frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1}+C=$

$=\frac{x^{\frac{4}{3}}}{\frac{4}{3}}+C=\frac{3}{4}\sqrt[3]{x^4}+C$

2.9 $\int x^{-10}dx=\frac{x^{-10+1}}{-10+1}+C=\frac{x^{-9}}{-9}+C=-\frac{1}{9x^9}+C$

2.10 $\int (x^5+x^4)dx=\int x^5dx+\int x^4dx=$

$=\frac{x^{5+1}}{5+1}+\frac{x^{4+1}}{4+1}+C=\frac{x^6}{6}+\frac{x^5}{5}+C$

2.11 $\int \left(8x^2-7x+\frac{1}{2}\right)dx=$

$=8\int x^2dx-7\int xdx+\frac{1}{2}\int dx=$

$=8\frac{x^3}{3}-7\frac{x^2}{2}+\frac{1}{2}x+C=$

$=\frac{8}{3}x^3-\frac{7}{2}x^2+\frac{1}{2}x+C$

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3.1 $\int x^{10}dx=\frac{x^{10+1}}{10+1}+C=\frac{x^{11}}{11}+C$

3.2 $\int \frac{1}{\sqrt{x^3}}dx=\int x^{-\frac{3}{2}}dx=\frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1}+C=$

$=\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}+C=-\frac{2}{\sqrt{x}}+C$

3.3 $\int \frac{5}{\sqrt{5x^3}}dx=\frac{5}{\sqrt{5}}\int x^{-\frac{3}{2}}dx=\sqrt{5}\left(-\frac{2}{\sqrt{x}}\right)+C=$

$=-\frac{2\sqrt{5}}{\sqrt{x}}+C$

3.4 $\int (e^x+3^x-\sin x+3)dx=$

$=\int e^xdx+\int 3^xdx+\int -\sin xdx+3\int dx=$

$=e^x+\frac{3^x}{\ln 3}+\cos x+3x+C$

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4.1 $\int x\ln xdx$

Seja $u=\ln x \Rightarrow du=\frac{1}{x}dx$

$dv=xdv \Rightarrow v=\frac{x^2}{2}$

$\int x\ln xdx=\frac{x^2}{2}\ln x-\int\left(\frac{x^2}{2}\cdot\frac{1}{x}\right)dx=$

$=\frac{x^2}{2}\ln x-\frac{1}{2}\int xdx=$

$=\frac{x^2}{2}\ln x-\frac{1}{2}\frac{x^2}{2}+C=$

$=\frac{x^2}{2}\ln x-\frac{x^2}{4}+C$

4.2 $\int xe^{2x}dx$

Seja $u=x \Rightarrow du=dx$

$dv=e^{2x}dv \Rightarrow v=\frac{1}{2}e^{2x}$

$\int xe^{2x}dx=\frac{1}{2}xe^{2x}-\int \frac{1}{2}e^{2x}dx=$

$=\frac{1}{2}xe^{2x}-\frac{1}{2}\int e^{2x}dx=$

$=\frac{1}{2}x^{2x}-\frac{1}{2}\left(\frac{1}{2}e^{2x}\right)+C=$

$=\frac{1}{2}xe^{2x}-\frac{1}{4}e^{2x}+C$

4.3 $\int e^x \cos x dx$

Seja $u = \cos x \Rightarrow du = -\sin x dx$

$dv = e^x dv \Rightarrow v = e^x$

$\int e^x \cos x dx = e^x \cos x - \int (-e^x \sin x) dx =$

$= e^x \cos x + \int e^x \sin x dx$

Seja $u = \sin x \Rightarrow du = \cos x dx$

$dv = e^x dv \Rightarrow v = e^x$

$\int e^x \cos x dx = e^x \cos x + [e^x \sin x - (e^x \cos x) dx] =$

$= e^x \cos x + e^x \sin x - \int e^x \cos x dx$

Considerando $I = \int e^x \cos x dx$, vem:

$I = e^x \cos x + e^x \sin x - I$ ou seja:

$2I = e^x \cos x + e^x \sin x$

Logo,

$I = \frac{1}{2}(e^x \cos x + e^x \sin x) + C$

$\int e^x \cos x dx = \frac{1}{2} e^x (\cos x + \sin x) + C$

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1. $\int 3x^5 dx = 3\int x^5 dx =$

$= 3\frac{x^{5+1}}{5+1} + C = 3\frac{x^6}{6} + C = \frac{1}{6}x^6 + C$

(A)

2. $\int (3x^3 - 5x^2 + 3) dx =$

$= 3\int x^3 dx - 5\int x^2 dx + 3\int dx =$

$= 3\frac{x^4}{4} - 5\frac{x^3}{3} + 3x + C$

$= \frac{3}{4}x^4 - \frac{5}{3}x^3 + 3x + C$

(C)

3. $\int \frac{\sqrt{2x}}{\sqrt[3]{5x}} dx = \frac{\sqrt{2}}{\sqrt[3]{5}} \int \frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} dx =$

$= \int \frac{\sqrt{2}}{\sqrt[3]{5}} \int x^{\frac{1}{6}} dx = \frac{\sqrt{2}}{\sqrt[3]{5}} \times \frac{x^{\frac{1}{6}+1}}{\frac{1}{6}+1} + C =$

$= \frac{\sqrt{2}}{\sqrt[3]{5}} \times \frac{6}{7} + C = \frac{6\sqrt{2}\sqrt[6]{x^7}}{7\sqrt[3]{5}} + C$

(B)

4. $\int x^2 e^{2x} dx$

Seja $u = x^2 \Rightarrow du = 2x dx$

$dv = e^{2x} dx \Rightarrow v = \frac{1}{2}e^{2x}$

$\int x^2 e^{2x} dx = \frac{1}{2}x^2 e^{2x} - \int \left(\frac{1}{2}e^{2x} \times 2x\right) dx =$

$= \frac{1}{2}x^2 e^{2x} - \int x e^{2x} dx$

Seja $u = x \Rightarrow du = dx$

$dv = e^{2x} dx \Rightarrow v = \frac{1}{2}e^{2x} dx$

$\int x^2 e^{2x} dx = \frac{1}{2}x^2 e^{2x} - \left[\frac{1}{2}x e^{2x} - \int \frac{1}{2}e^{2x} dx\right] =$

$= \frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{2}\int e^{2x} dx =$

$= \frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{2}\left(\frac{1}{2}e^{2x}\right) + C =$

$= e^{2x}\left(\frac{1}{2}x^2 - \frac{1}{2}x + \frac{1}{4}\right) + C =$

(D)

5. $\int x^2 \sin x dx$

Seja $u = x^2 \Rightarrow du = 2x dx$

$dv = \sin x dv \Rightarrow v = -\cos x$

$\int x^2 \sin x dx = -x^2 \cos x - \int (-2x \cos x) dx =$

$= -x^2 \cos x + 2\int x \cos x dx$

Seja $u = x \Rightarrow du = dx$

$dv = \cos x dx \Rightarrow v = \sin x$

$\int x^2 \sin x dx = -x^2 \cos x + 2[x \sin x - \int \sin x dx] =$

$= -x^2 \cos x + 2(x \sin x + \cos x) + C =$

$= -x^2 \cos x + 2x \sin x + 2\cos x + C$

(A)

6. $\int \frac{\ln x}{x^2} dx$

Seja $u = \ln x \Rightarrow du = \frac{1}{x} dx$

$dv = \frac{1}{x^2} dx \Rightarrow dv = x^{-2} dx \Rightarrow v = -x^{-1}$

$$\int \frac{\ln x}{x^2} dx = -x^{-1} \ln x - \int \left(-x^{-1} \times \frac{1}{x} \right) dx =$$
$$= -\frac{\ln x}{x} + \int x^{-2} dx =$$
$$= -\frac{\ln x}{x} - x^{-1} + C =$$
$$= -\frac{\ln x}{x} - \frac{1}{x} + C$$

(B)

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1.1 $\int x^2 dx = \frac{x^{2+1}}{2+1} + C = \frac{1}{3} x^3 + C$

1.2 $\int x^7 dx = \frac{x^{7+1}}{7+1} + C = \frac{1}{8} x^8 + C$

1.3 $\int x dx = \frac{x^{1+1}}{1+1} + C = \frac{1}{2} x^2 + C$

1.4 $\int \sqrt{x} dx = x^{\frac{1}{2}} dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{2}{3} \sqrt{x^3} + C$

1.5 $\int 3x^5 dx = 3 \int x^5 dx = 3 \frac{x^{5+1}}{5+1} + C = \frac{1}{2} x^6 + C$

1.6 $\int \frac{7}{x^3} dx = 7 \int x^{-3} dx = 7 \frac{x^{-3+1}}{-3+1} + C =$

$$= -\frac{7}{2} x^{-2} + C = -\frac{7}{x^2} + C$$

1.7 $\int \sqrt{x^3} dx = \int x^{\frac{3}{2}} dx = \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C =$

$$= \frac{2}{5} x^{\frac{5}{2}} + C = \frac{2}{5} \sqrt{x^5} + C$$

1.8 $\int \frac{1}{\sqrt[3]{x^2}} dx = \int x^{-\frac{2}{3}} dx = \frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} + C =$

$$= 3x^{\frac{1}{3}} + C = 3\sqrt[3]{x} + C$$

1.9 $\int \frac{5x^5 - 2x^4 + 3x^2 + 4}{x^2} dx =$

$$= \int \left(5x^3 - 2x^3 + 3 + 4x^{-2}\right) dx =$$
$$= 5 \int x^3 dx - 2 \int x^2 dx + 3 \int dx + 4 \int x^{-2} dx =$$
$$= \frac{5}{4} x^4 - \frac{2}{3} x^3 + 3x - \frac{4}{x} + C$$

1.10 $\int \sin x dx = -\cos x + C$

1.11 $\int \cos x dx = \sin x + C$

1.12 $\int \left(1 + \text{tg}^2 x\right) dx = \int \frac{1}{\cos^2 x} dx = \text{tg}\, x + C$

1.13 $\int \frac{1}{1+x^2} dx = \text{arc tg}\, x + C$

1.14 $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$

1.15 $\int \frac{-1}{\sqrt{1-x^2}} dx = \arccos x + C$

1.16 $\int e^x dx = e^x + C$

1.17 $\int a^x dx = \frac{a^x}{\ln a} + C$

2.1 $\int x e^x dx$

Seja $u = x \Rightarrow du = dx$

$dv = e^x dx \Rightarrow v = e^x$

$$\int x e^x dx = x e^x - \int e^x dx =$$
$$= x e^x - e^x + C =$$
$$= e^x (x - 1) + C$$

2.2 $\int x (x+3)^3 dx$

Seja $u = x \Rightarrow du = dx$

$dv = (x+3)^3 dx \Rightarrow v = \frac{1}{4} (x+3)^4$

$$\int x (x+3)^3 dx = \frac{1}{4} x (x+3)^4 - \int \frac{1}{4} (x+3)^4 dx =$$
$$= \frac{1}{4} x (x+3)^4 - \frac{1}{4} \left(\frac{1}{5} (x+3)^5 \right) + C =$$
$$= \frac{1}{4} x (x+3)^4 - \frac{1}{20} (x+3)^5 + C =$$

$$=\frac{1}{4}(x+3)^4\left(x-\frac{1}{5}(x+3)\right)+C=$$

$$=\frac{1}{4}(x+3)^4\left(\frac{4}{5}x-\frac{3}{5}\right)+C=$$

$$=\frac{1}{20}(x+3)^4(4x-3)+C$$

2.3 $\int x^3 \ln x dx$

Seja $u=\ln x \Rightarrow du=\frac{1}{x}dx$

$dv=x^3dx \Rightarrow v=\frac{1}{4}x^4$

$$\int x^3 \ln dx=\frac{1}{4}x^4\ln x-\int\frac{1}{4}x^4\times\frac{1}{x}dx=$$

$$=\frac{1}{4}x^4\ln x-\frac{1}{4}x^3dx=$$

$$=\frac{1}{4}x^4\ln x-\frac{1}{16}x^4+C=$$

$$=\frac{1}{16}x^4(4\ln x-1)+C=$$

$$=\frac{x^3}{16}(4\ln x-1)+C$$

2.4 $\int x^5 \ln x dx$

Seja $u=\ln x \Rightarrow du=\frac{1}{x}dx$

$dv=x^5dx \Rightarrow v=\frac{x^6}{6}$

$$\int x^5 \ln dx=\frac{x^6}{6}\ln x-\int\frac{x^6}{6}\times\frac{1}{x}dx=$$

$$=\frac{x^6}{6}\ln x-\frac{1}{6}\int x^5dx=$$

$$=\frac{x^6}{6}\ln x-\frac{1}{36}x^6+C$$

2.5 $\int x^2 \ln x dx$

Seja $u=\ln x \Rightarrow du=\frac{1}{x}dx$

$dv=x^2dx \Rightarrow v=\frac{x^3}{3}$

$$\int x^2 \ln dx=\frac{x^3}{3}\ln x-\int\left(\frac{x^3}{3}\times\frac{1}{x}\right)dx=$$

$$=\frac{x^3}{3}\ln x-\int\frac{x^2}{3}dx=$$

$$=\frac{x^3}{3}\ln x-\frac{x^3}{9}+C$$