PROPOSTAS DE RESOLUÇÃO

Capítulo 1

Pág. 8

1.1 $\left|-2-(-3)\right|^2 = \left|-2+3\right|^2 = 1^2 = 1$

1.2 $\left|-1\right|^{\frac{1}{2}} + \left|-(-1)\right|^2 = 1^{\frac{1}{2}} + 1^2 + 1^2 = \sqrt{1}+1 = 2$

1.3 $\left|-1-3\right|^2 - \left|-(-4)\right|^{\frac{1}{2}} = \left|-4\right|^2 - 4^{\frac{1}{2}} = 4^2 - \sqrt{4} = 14$

Pág. 9

2.1 $|x-1| = 3 \Leftrightarrow x-1=3 \vee x-1=-3$

$\Leftrightarrow x = 4 \vee x = -2$

$S = \{-2, 4\}$

2.2 $\left|-\frac{1}{2}x-1\right| = \frac{1}{3} \Leftrightarrow -\frac{1}{2}x-1 = \frac{1}{3} \vee -\frac{1}{2}x-1 = -\frac{1}{3}$

$\Leftrightarrow -\frac{1}{2}x = \frac{4}{3} \vee -\frac{1}{2}x = \frac{2}{3}$

$\Leftrightarrow x = -\frac{8}{3} \vee x = -\frac{4}{3}$

$S = \left\{-\frac{8}{3}, -\frac{4}{3}\right\}$

2.3 $\left|-2x+\frac{1}{2}x^2\right| = 0 \Leftrightarrow -2x+\frac{1}{2}x^2 = 0 \Leftrightarrow x\left(\frac{1}{2}x-2\right) = 0$

$\Leftrightarrow x = 0 \vee \frac{1}{2}x-2 = 0 \Leftrightarrow x = 0 \vee x = 4$

$S = \{0, 4\}$

2.4 $|-1-2x| = -1 \Leftrightarrow x \in \varnothing$

$S = \{\}$

2.5 $|2x-1| > 0 \Leftrightarrow 2x-1 \neq 0 \Leftrightarrow x \neq \frac{1}{2}$

$S = \mathbb{R} \setminus \left\{\frac{1}{2}\right\}$

2.6 $|2x-1| < 3 \Leftrightarrow 2x-1 < 3 \wedge 2x-1 > -3$

$\Leftrightarrow 2x < 4 \wedge 2x > -2 \Leftrightarrow x < 2 \wedge x > -1$

$\Leftrightarrow x \in \left]-1, 2\right[$

$S = \left]-1, 2\right[$

2.7 $\left|-2x+\frac{1}{2}\right| > 5 \Leftrightarrow -2x+\frac{1}{2} > 5 \vee -2x+\frac{1}{2} < -5$

$\Leftrightarrow -4x+1 > 10 \vee -4x+1 < -10$

$\Leftrightarrow -4x > 9 \vee -4x < -11$

$\Leftrightarrow x < -\frac{9}{4} \vee x > \frac{11}{4}$

$S = \left]-\infty, -\frac{9}{4}\right[\cup \left]\frac{11}{4}, +\infty\right[$

2.8 $\left|-x+\frac{1}{2}\right| + 2 > 7 \Leftrightarrow \left|-x+\frac{1}{2}\right| > 5$

$\Leftrightarrow -x+\frac{1}{2} > 5 \vee -x+\frac{1}{2} < -5$

$\Leftrightarrow -x > \frac{9}{2} \vee -x < -\frac{11}{2}$

$\Leftrightarrow x < -\frac{9}{2} \vee x > \frac{11}{2}$

$S = \left]-\infty, -\frac{9}{2}\right[\cup \left]\frac{11}{2}, +\infty\right[$

2.9 $|-2x+3| < -5 \Leftrightarrow x \in \varnothing$

$S = \{\}$

2.10 $|-3x+1| > -8 \Leftrightarrow x \in \mathbb{R}$

$S = \mathbb{R}$

Pág. 10

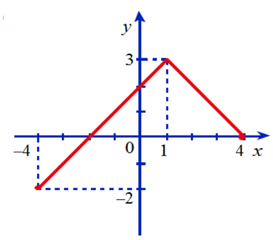
3.1 **a)** $f(x) = \begin{cases} -(x-1)+3 & \text{se } x-1 \geq 0 \wedge x \in [-4, 4] \\ -[-(x-1)]+3 & \text{se } x-1 < 0 \wedge x \in [-4, 4] \end{cases}$

$f(x) = \begin{cases} -x+4 & \text{se } x \in [1, 4] \\ x+2 & \text{se } x \in [-4, 1[\end{cases}$

b)

x	$y = -x+4$
1	3
4	0

x	$y = x+2$
– 4	– 2
1	3



c) $D'_f = [-2, 3]$

d) Zeros: – 2 e 4.

Monotonia: f é estritamente crescente em $[-4, 1]$ e é estritamente decrescente em $[1, 4]$.

Pág.11

3.2 **a)** $|x-1|-1 = \begin{cases} x-1-1 & \text{se } x-1 \geq 0 \\ -x+1-1 & \text{se } x-1 < 0 \end{cases}$

$\Leftrightarrow |x-1|-1 = \begin{cases} x-2 & \text{se } x \geq 1 \\ -x & \text{se } x < 1 \end{cases}$

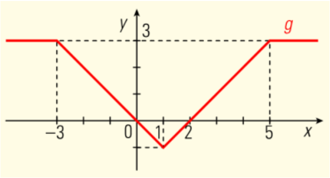
Logo,

$g(x) = \begin{cases} -x & \text{se } -3 \leq x < 1 \\ x-2 & \text{se } 1 \leq x \leq 5 \\ 3 & \text{se } x < -3 \vee x > 5 \end{cases}$

Gráficos:

x	$y = -x$
– 3	3
1	– 1

x	$y = x - 2$
1	– 1
5	3



b) Zeros: 0 e 2.

Monotonia: g é estritamente decrescente em [– 3, 1], estritamente crescente em [1, 5] e constante em $]-\infty,-3]$ e em $[5,+\infty[$.

4.1 $f(x) = -|x-3| + 5$

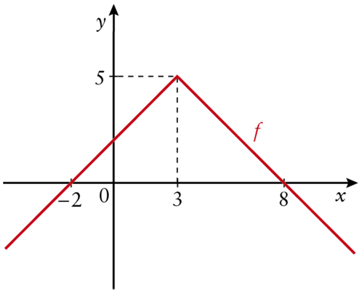
$$f(x)=\begin{cases}-x+8 & \text{se } x\geq 3\\ x+2 & \text{se } x<3\end{cases}$$

$y=-x+8$

x	y
3	5
8	0

$y=x+2$

x	y
3	5
– 2	0



4.2. $g(x)=-\left|\frac{1}{2}x\right|-3$

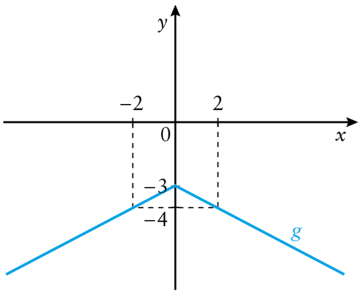
$$g(x)=\begin{cases}-\frac{1}{2}x-3 & \text{se } x\geq 0\\ \frac{1}{2}x-3 & \text{se } x<0\end{cases}$$

$y=-\frac{1}{2}x-3$

x	y
0	– 3
2	– 4

$y=\frac{1}{2}x-3$

x	y
0	– 3
2	– 2



4.3. $h(x) = -2|x| + 3$

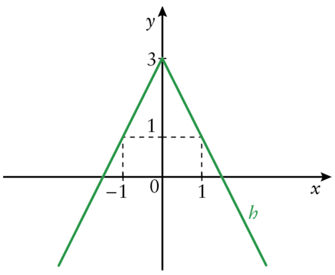
$$h(x)=\begin{cases}-2x+3 & \text{se } x\geq 0\\ 2x+3 & \text{se } x<0\end{cases}$$

$y=-2x+3$

x	y
0	3
1	1

$y=2x+3$

x	y
0	3
– 1	1



4.4 $i(x) = -\ |2x-3| + 1$

$$i(x)=\begin{cases} -(2x-3)+1 & \text{se } x \geq \frac{3}{2} \\ -(-2x+3)+1 & \text{se } x < \frac{3}{2} \end{cases}$$

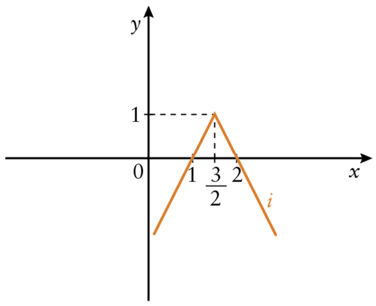
$$i(x)=\begin{cases} -2x+4 & \text{se } x \geq \frac{3}{2} \\ 2x-2 & \text{se } x < \frac{3}{2} \end{cases}$$

$y = 2x+4$

x	y
$\frac{3}{2}$	1
2	0

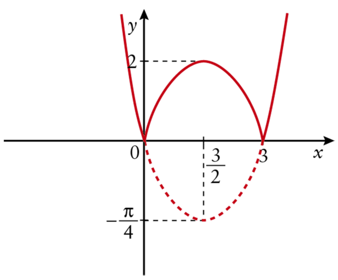
$y = 2x-2$

x	y
$\frac{3}{2}$	1
1	0

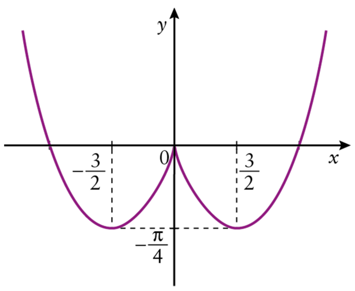


Pág. 13

5.1 **a)**



b)



5.2 Pode, por exemplo a função definida, em IR, por $f(x) = x^4 - x^2$.
f tem três zeros: -1, 0 e 1 e é par,
$f(-x) = (-x)^4 - (-x)^2 = x^4 - x^2 = f(x)$, qualquer que seja o valor real de x.

6.1 $|x-3| = |x-1| \Leftrightarrow$

$x-3 = x-1 \vee x-3 = -(x-1) \Leftrightarrow$

$0x = 2 \vee x-3 = -x+1 \Leftrightarrow$

$2x = 4 \Leftrightarrow x = 2$

$S = \{2\}$

6.2 $2|x-3| = -3|2x-1| \Leftrightarrow$

$|2x-6| = -|6x-3| \Leftrightarrow$

$|2x-6| + |6x-3| = 0$

A soma de dois números não negativos, $|2x-6|$ e $|6x-3|$, só é nulo se os dois números o forem simultaneamente, ou seja:

$|2x-6| = 0 \wedge |6x-3| = 0 \Leftrightarrow$

$2x-6 = 0 \wedge 6x-3 = 0 \Leftrightarrow$

$x = 3 \wedge x = \frac{1}{2}$, impossível.

$S = \{\}$

7.1 $2|x-3| \geq 3|x+1| \Leftrightarrow |2x-6| - |3x+3| \geq 0$

Seja $f(x) = |2x-6| - |3x+3| \Leftrightarrow$

$$\Leftrightarrow f(x)=\begin{cases} x+9 & \text{se } x \leq -1 \\ -5x+3 & \text{se } -1 < x < 3 \\ -x-9 & \text{se } x \geq 3 \end{cases}$$

$y = x+9$

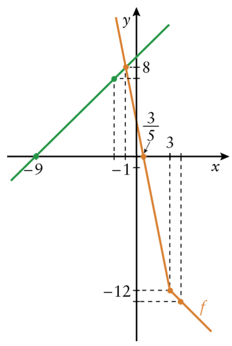
x	y
−1	8
−2	7

$y = -5x+3$

x	y
−1	8
3	−12

$y = -x-9$

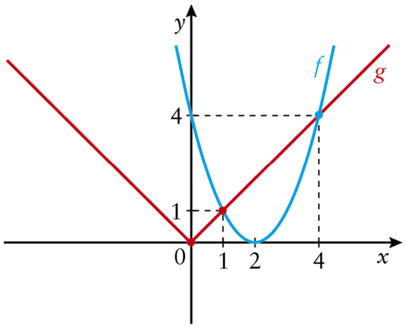
x	y
3	−12
4	−13



$f(x) \geq 0 \Leftrightarrow x \in \left[-9, \frac{3}{5}\right]$

7.2 $|x-2|^2-|x|\le 0 \Leftrightarrow (x-2)^2 \le |x|$

Seja $f(x)=(x-2)^2$ e $g(x)=|x|$



$|x-2|^2-|x|\le 0 \Leftrightarrow |x-2|^2 \le |x| \Leftrightarrow f(x)\le g(x) \Leftrightarrow$

$x\in[1,\ 4]$

Pág. 14

1. $f(x) = |1-x| - 4$; $g(x) = |f(x)|$

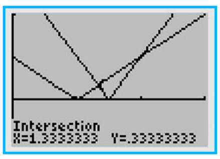
$g(2) = |f(2)| = ||1-2|-4| = |1-4| = 3$

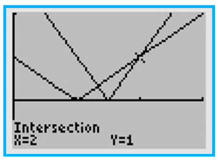
Resposta: (D)

2. $D'_f = [-1,\ 2]$

Resposta: (B)

3.

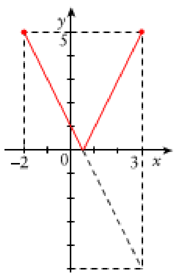




4. $f(x) = -2x + 1$; $D'_f = [-2,\ 3]$

$f(-2) = -2\times(-2) + 1 = 5$

$f(3) = -2\times 3 + 1 = -5$



Resposta: (C)

Pág. 15

1. $A(-3,\ 4)$; $B(0,\ 1)$ e $C(2,\ 3)$.

$m=\dfrac{4-1}{-3-0}=-1$

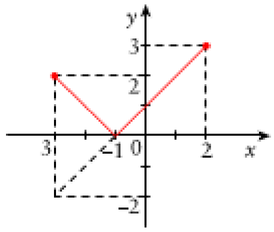
$y-1=-1(x-0)\Leftrightarrow y=-x+1$

$m=\dfrac{1-3}{0-2}=1$

$y-1=-1(x-0)\Leftrightarrow y=x+1$

2.1 $f(x)=\begin{cases}2 & \text{se } -9\le x\le -3\\ -x+1 & \text{se } -3\le x\le 0\\ x+1 & \text{se } 0< x\le 2\\ 3 & \text{se } 2< x\le 9\end{cases}$

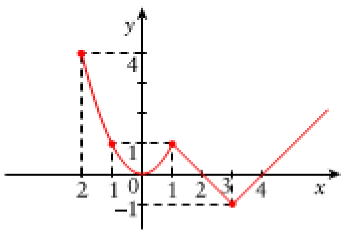
x	$x+1$
-3	-2
2	3



2.2 $g(x)=\begin{cases}|x-3|-1 & \text{se } x>1\\ x^2 & \text{se } -2\le x\le 1\end{cases}$

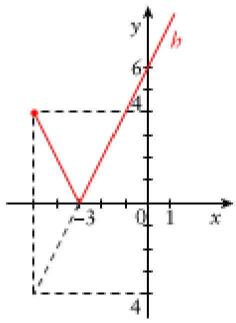
$g(x)=\begin{cases}x-3-1 & \text{se } x\ge 3\wedge x>1\\ -x+3-1 & \text{se } x<3\wedge x>1\\ x^2 & \text{se } -2\le x\le 1\end{cases}$

$g(x)=\begin{cases}x-4 & \text{se } x\ge 3\\ -x+2 & \text{se } 1<x<3\\ x^2 & \text{se } -2\le x\le 1\end{cases}$



3.1 $h(x)=2|x+3|$, $x\in[-5,+\infty[$

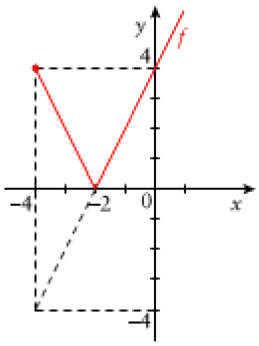
x	$y = 2x + 6$
-5	-4
-3	0
0	6



3.2 **a)** $h(x-1)=2|x-1+3| \wedge x-1 \geq -5$

$f(x)=2|x+2| \wedge x \geq -4$

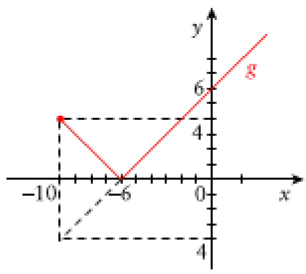
x	$y = 2x + 4$
– 4	– 4
– 2	0
0	4



b) $h\left(\frac{x}{2}\right)=2\left|\frac{x}{2}+3\right| \wedge \frac{x}{2} \geq -5$

$g(x)=|x+6| \wedge x \geq -10$

x	$y = x + 6$
– 10	– 4
– 6	0
0	6



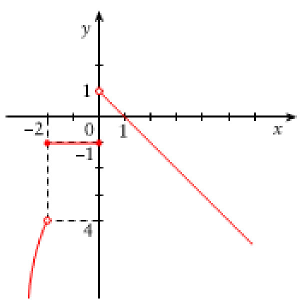
4. $D_h=[-5,+\infty[$; $D'_h=[0,+\infty[$; Zeros: $\{-3\}$; h é decrescente em [– 5, – 3] e crescente em $[-3, +\infty[$.

$D_f=[-4,+\infty[$; $D'_f=[0,+\infty[$; Zeros: {– 2}; f é decrescente em [– 4, – 2] e crescente em $[-2,+\infty[$.

$D_g=[-10,+\infty[$; $D'_g=[0,+\infty[$; Zeros: $\{-6\}$; g é decrescente em [– 10, – 6] e crescente em $[-6,+\infty[$.

5. $f(x)=\begin{cases} -x+1 & \text{se } x>0 \\ -1 & \text{se } -2 \leq x \leq 0 \\ -x^2 & \text{se } x<-2 \end{cases}$

5.1



5.2 $D'_f=]-\infty,1[$

5.3 $f(x)=0 \Leftrightarrow x=1$

Resposta: {1}

5.4 $D'_g=[0,+\infty[$

5.5 $h(x)=-|3x|=\begin{cases} -3x & \text{se } x>0 \\ 3x & \text{se } x \leq 0 \end{cases}$

Para $x < -2$,

$f(x)=h(x) \Leftrightarrow -x^2=3x \wedge x<-2$

$\Leftrightarrow 3x+x^2=0 \wedge x<-2$

$\Leftrightarrow x(3+x)=0 \wedge x<-2$

$\Leftrightarrow x=-3$

$f(-3)=-9$

Para $-2 \leq x \leq 0$,

$f(x)=h(x) \Leftrightarrow 3x=-1 \wedge -2 \leq x \leq 0 \Leftrightarrow x=-\frac{1}{3}$

$f\left(-\frac{1}{3}\right)=-1$

Para $x > 0$,

$f(x)=h(x) \Leftrightarrow -x+1=-3x \wedge x>0$

$\Leftrightarrow 2x=-1 \wedge x>0$

$\Leftrightarrow x \in \Phi$

Resposta: (– 3, – 9) e $\left(-\frac{1}{3},-1\right)$.

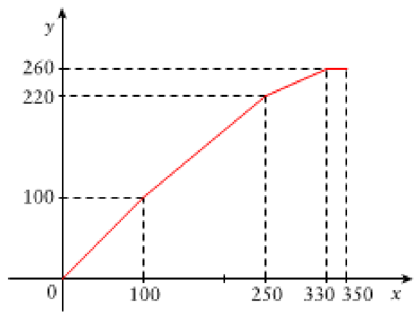
6. 100 km : 100 × 1 Mt = 100 Mt

250 km : 100 Mt + 150 × 0,8 Mt = 220 Mt

330 km : 220 Mt + 80 × 0,5 Mt = 260 Mt

350 km : 260 Mt

6.1 $f(x)=\begin{cases} x & \text{se } 0 \leq x \leq 100 \\ 100+0{,}8(x-100) & \text{se } 100<x \leq 250 \\ 220+0{,}5(x-250) & \text{se } 250<x \leq 330 \\ 260 & \text{se } 330<x \leq 350 \end{cases}$



6.2 f (300) = 200 + 0,5 (300 – 250) = 245 Mt

3 × 245 Mt = 735 Mt

Resposta: 735 Mt

6.3 f (175) = 100 + 0,8 × (175 – 100) = 100 + 0,8 × 75

= 160 Mt ← custo de uma tonelada.

Logo, C (x) = 160x (meticais).