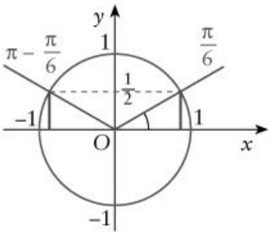
1.1. **a)** $\sin x = \frac{1}{2} \Leftrightarrow \sin x = \sin\frac{\pi}{6}$ **Pág. 220**

$\Leftrightarrow x = \frac{\pi}{6} + 2k\pi \vee x = \left(\pi - \frac{\pi}{6}\right) + 2k\pi,\ k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{6} + 2k\pi \vee x = \frac{5\pi}{6} + 2k\pi,\ k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{6} + 2k\pi \vee x = \frac{5\pi}{6} + 2k\pi,\ k \in \mathbb{Z}$

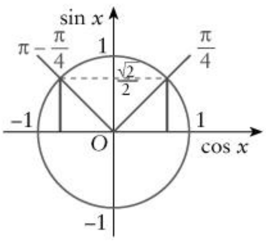


b) $2\sin x - \sqrt{2} = 0 \Leftrightarrow 2\sin x = \sqrt{2}$

$\Leftrightarrow \sin x = \frac{\sqrt{2}}{2} \Leftrightarrow \sin x = \sin\frac{\pi}{4}$

$\Leftrightarrow x = \frac{\pi}{4} + 2k\pi \vee x = \pi - \frac{\pi}{4} + 2k\pi,\ k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{4} + 2k\pi \vee x = \frac{3\pi}{4} + 2k\pi,\ k \in \mathbb{Z}$

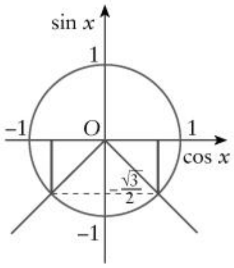


c) $-\sqrt{3} = 2\sin x \Leftrightarrow \sin x = -\frac{\sqrt{3}}{2}$

$\Leftrightarrow \sin x = \sin\left(-\frac{\pi}{3}\right)$

$\Leftrightarrow x = -\frac{\pi}{3} + 2k\pi \vee x = \pi + \frac{\pi}{3} + 2k\pi,\ k \in \mathbb{Z}$

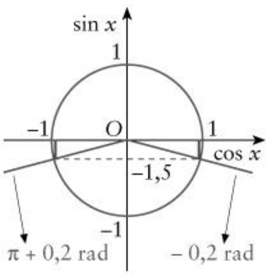
$\Leftrightarrow x = -\frac{\pi}{3} + 2k\pi \vee x = \frac{4\pi}{3} + 2k\pi,\ k \in \mathbb{Z}$



d) $\frac{1}{5} + \sin x = 0 \Leftrightarrow \sin x = -\frac{1}{5}$

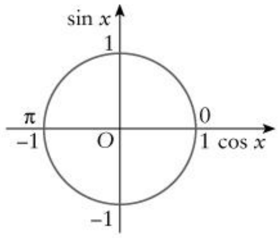
$\Leftrightarrow \sin x = \sin(-0{,}2)$

$\Leftrightarrow x = -0{,}2 + 2k\pi \vee x = (\pi + 0{,}2) + 2k\pi,\ k \in \mathbb{Z}$



1.2 $f: x \frown y = \sin(2x)$

a)



$f(x) = 0 \Leftrightarrow \sin(2x) = 0 \Leftrightarrow \sin(2x) = \sin 0$

$\Leftrightarrow 2x = 0 + k\pi,\ k \in \mathbb{Z} \Leftrightarrow x = \frac{\pi}{2}k,\ k \in \mathbb{Z}$

b) Valores de $x \in [-\pi, \pi]$: $f(x)$ seja máximo relativo.

$f(x) = \sin(2x) \quad -1 \leqslant \sin(2x) \leqslant 1$

O valor máximo da função é 1, logo, temos de descobrir que valores de $x \in [-\pi, \pi]$ transformam a função em 1.

$f(x) = 1 \Leftrightarrow \sin(2x) = 1$

$\Leftrightarrow \sin(2x) = \sin\frac{\pi}{2}$

$\Leftrightarrow 2x = \frac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{4} + k\pi,\ k \in \mathbb{Z}$

Como $x \in [-\pi, \pi]$, apenas temos as soluções:

$k = -1: x = \frac{\pi}{4} - \pi \Leftrightarrow x = \frac{\pi}{4} - \frac{4\pi}{4}$

$\Leftrightarrow x = -\frac{3\pi}{4} \in [-\pi, \pi]$

$k = 0: \quad x = \frac{\pi}{4} \in [-\pi, \pi]$

$k = 1: \quad x = \frac{\pi}{4} + \underset{(4)}{\frac{\pi}{1}} \Leftrightarrow x = \frac{5\pi}{4} \notin [-\pi, \pi]$

R.: Logo no intervalo $[-\pi, \pi]$, temos como solução para os valores de x: $-\frac{3\pi}{4}$ e $\frac{\pi}{4}$.

2.1 $\sin(3x) = 1 \Leftrightarrow \sin(3x) = \sin\frac{\pi}{2}$ **Pág. 221**

$\Leftrightarrow 3x = \frac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{6} + \frac{2}{3}k\pi,\ k \in \mathbb{Z}$

2.2 $\sin(-2x) = 0 \Leftrightarrow \sin(-2x) = \sin 0$

$\Leftrightarrow -2x = 0 + k\pi,\ k \in \mathbb{Z}$

$\Leftrightarrow x = -\frac{k\pi}{2},\ k \in \mathbb{Z}$

2.3 $\sin^2 x = \sin x \Leftrightarrow \sin^2 x - \sin x = 0$

$\Leftrightarrow \sin x(\sin x - 1) = 0$

$\Leftrightarrow \sin x = 0 \vee \sin x = 1$

$\Leftrightarrow \sin x = \sin 0 \vee \sin x = \sin\frac{\pi}{2}$

$\Leftrightarrow x = k\pi \vee x = \frac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z}$

3.1 a) $\sin(-2x) + \sin x = 0$ **Pág. 222**

$\Leftrightarrow \sin(-2x) = -\sin x$

$\Leftrightarrow \sin(-2x) = \sin(-x)$

$\Leftrightarrow -2x = -x + 2k\pi \vee -2x = \pi - (-x) + 2k\pi,\ k \in \mathbb{Z}$

$\Leftrightarrow -x = 2k\pi \vee -2x = \pi + x + 2k\pi,\ k \in \mathbb{Z}$

$\Leftrightarrow x = -2k\pi \vee -3x = \pi + 2k\pi,\ k \in \mathbb{Z}$

$\Leftrightarrow x = -2k\pi \vee x = -\frac{\pi}{3} - \frac{2}{3}k\pi,\ k \in \mathbb{Z}$

$k=-1: x = 2\pi \ \vee \ x = -\frac{\pi}{3} + \frac{2}{3}\pi$

$x = \cancel{2\pi} \ \vee \ x = \frac{\pi}{3}$

$k=-2: x = 4\pi \ \vee \ x = -\frac{\pi}{3} - \frac{2}{3} \times (-2)\pi$

$x = \cancel{4\pi} \ \vee \ x = \pi$

$k=-3: x = 6\pi \ \vee \ x = -\frac{\pi}{3} - \frac{2}{3} \times (-3)\pi$

$x = 6\pi \ \vee \ x = -\frac{\pi}{3} + \frac{6\pi}{3}$

$x = \cancel{6\pi} \ \vee \ x = \frac{5\pi}{3}$

$k=0: \quad x = 0 \ \vee \ x = -\frac{\pi}{3}$

$k=1: \quad x = -2\pi \ \vee \ x = -\frac{\pi}{3} - \frac{2}{3\pi}$

$x = -\cancel{2\pi} \ \vee \ x = -\pi$

$k=2: \quad x = -2 \times 2\pi \ \vee \ x = -\frac{\pi}{3} - \frac{2}{3} \times 2\pi$

$x = -4\pi \ \vee \ x = -\frac{\pi}{3} - \frac{4\pi}{3}$

$x = -\cancel{4\pi} \ \vee \ x = -\frac{5\pi}{3}$

$k=3: \quad x = -6\pi \ \vee \ x = -\frac{\pi}{3} - \frac{2}{3} \times 3\pi$

$x = -6\pi \ \vee \ x = -\frac{\pi}{3} - \frac{6\pi}{3}$

$x = -\cancel{6\pi} \ \vee \ x = -\cancel{\frac{7\pi}{3}}$

Logo no intervalo $]-360°, 360°[$ a equação $\sin(-2x) + \sin x = 0$ tem as seguintes soluções: $-300°$; $-180°$; $-60°$; $0°$; $60°$; $180°$; $300°$.

3.1 b) $2\sin\left(\frac{x}{3}\right) = 1 \iff \sin\frac{x}{3} = \frac{1}{2}$

$\iff \sin\frac{x}{3} = \sin 30°$

$\iff \frac{x}{3} = 30° + 360°\,k \ \vee \ \frac{x}{3} = (180° - 30°) + {}$
$+ 360°\,k, \ k \in \mathbb{Z}$

$\iff x = 90° + 1080°\,k \ \vee \ \frac{x}{3} = 150° + 360°\,k,$
$k \in \mathbb{Z}$

$\iff x = 90° + 1080°\,k \ \vee \ x = 450° + 1080°\,k,$
$k \in \mathbb{Z}$

$k=0: \quad x = 90° \ \vee \ x = \cancel{450°}$

$k=1: \quad x = 90° + 1080° \ \vee \ x = 450° + 1080°$

$x = \cancel{1170°} \ \vee \ x = \cancel{1530°}$

$k=-1: x = 90° - 1080° \ \vee \ x = 450° - 1080°$

$x = -\cancel{990°} \ \vee \ x = -\cancel{630°}$

Logo no intervalo $]-360°, 360°[$ a equação $2\sin\left(\frac{x}{3}\right) = 1$ tem a seguinte solução: $90°$.

3.2 a) $\sin x = -\sin(2x) \iff \sin x = \sin(-2x)$

$\iff x = -2x + 2k\pi \ \vee \ x = \pi - (-2x) + 2k\pi,$
$k \in \mathbb{Z}$

$\iff 3x = 2k\pi \ \vee \ x = \pi + 2x + 2\pi k, \ k \in \mathbb{Z}$

$\iff x = \frac{2}{3}k\pi \ \vee \ -x = \pi + 2\pi k, \ k \in \mathbb{Z}$

$\iff x = \frac{2}{3}k\pi \ \vee \ x = -\pi - 2k\pi, \ k \in \mathbb{Z}$

$k=0: \quad x = 0 \ \vee \ x = -\cancel{\pi}$

$\iff x = -2x + 2k\pi \ \vee \ x = \pi - (-2x) + 2k\pi,$
$k \in \mathbb{Z}$

$\iff 3x = 2k\pi \ \vee \ x = \pi + 2x + 2\pi k, \ k \in \mathbb{Z}$

$\iff x = \frac{2}{3}k\pi \ \vee \ -x = \pi + 2\pi k, \ k \in \mathbb{Z}$

$\iff x = \frac{2}{3}k\pi \ \vee \ x = -\pi - 2k\pi, \ k \in \mathbb{Z}$

$k=0: \quad x = 0 \ \vee \ x = -\cancel{\pi}$

$k=1: \quad x = \frac{2\pi}{3} \ \vee \ x = -\pi - 2\pi$

$x = \frac{2\pi}{3} \ \vee \ x = -\cancel{3\pi}$

$k=2: \quad x = \frac{4\pi}{3} \ \vee \ x = -\pi - 4\pi$

$x = \frac{4\pi}{3} \ \vee \ x = -\cancel{5\pi}$

$k=3: \quad x = 2\pi \ \vee \ x = -\pi - 6\pi$

$x = 2\pi \ \vee \ x = -\cancel{7\pi}$

$k=-1: x = -\frac{2\pi}{3} \ \vee \ x = -\pi + 2\pi$

$x = -\cancel{\frac{2\pi}{3}} \ \vee \ x = \pi$

$k=-2: x = -\frac{4\pi}{3} \ \vee \ x = -\pi + 4\pi$

$x = -\cancel{\frac{4\pi}{3}} \ \vee \ x = \cancel{3\pi}$

Logo no intervalo $[0, 2\pi]$ a equação $\sin x = -\sin(2x)$ tem as seguintes soluções: 0; $\frac{2\pi}{3}$; π; $\frac{4\pi}{3}$ e 2π.

b) $\sin^2 x - \sin x = 0 \iff \sin x(\sin x - 1) = 0$

$\iff \sin x = 0 \ \vee \ \sin x = 1$

$\iff \sin x = \sin 0 \ \vee \ \sin x = \sin\frac{\pi}{2}$

$\iff x = k\pi \ \vee \ x = \frac{\pi}{2} + 2\pi k, \ k \in \mathbb{Z}$

$k=0: \quad x = 0 \ \vee \ x = \frac{\pi}{2}$

$k=1: \quad x = \pi \ \vee \ x = \cancel{\frac{\pi}{2} + 2\pi}$

$k=2: \quad x = 2\pi \ \vee \ x = \cancel{\frac{\pi}{2} + 4\pi}$

$k=-1: x = -\pi \ \vee \ x = \frac{\pi}{2} - \frac{2\pi}{\underset{(2)}{1}}$

$x = -\cancel{\pi} \ \vee \ x = -\cancel{\frac{3\pi}{2}}$

Logo no intervalo $[0, 2\pi]$ a equação $\sin^2 x - \sin x = 0$ tem as seguintes soluções: 0; $\frac{\pi}{2}$; π; 2π.

3.3 $f: x \mapsto y = \sin^2 x - 1$

$x \in [-\pi, \pi], \ f(x) = \sin\left(\frac{7\pi}{2}\right)$

$f(x) = \sin\left(\frac{7\pi}{2}\right) \iff \sin^2 x - 1 = \sin\left(\frac{7x}{2}\right)$

$\iff \sin^2 x - 1 = -1 \iff \sin^2 x = 0$

$\iff \sin x = 0 \iff \sin x = \sin 0$

$\iff x = \pi k, \ k \in \mathbb{Z}$

$k = -1 \longrightarrow x = -\pi$
$k = 0 \longrightarrow x = 0$
$k = 1 \longrightarrow x = \pi$

As soluções são $-\pi$, 0, π.

4. $f: x \curvearrowright y = 2\cos(2x)$, $x \in [-\pi, \pi]$ **Pág. 224**

4.1 $f(x) = 0 \Leftrightarrow 2\cos(2x) = 0 \Leftrightarrow \cos(2x) = 0$

$\Leftrightarrow \cos(2x) = \cos\frac{\pi}{2}$

$\Leftrightarrow 2x = \frac{\pi}{2} + 2k\pi \lor 2x = -\frac{\pi}{2} + 2k\pi\,,\ k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{4} + k\pi \lor x = -\frac{\pi}{4} + k\pi\,,\ k \in \mathbb{Z}$

$x \in [-\pi, \pi]$

$k = -1: x = \frac{\pi}{4} - \frac{4\pi}{4} \lor x = -\frac{\pi}{4} - \frac{4\pi}{4}$

$\Leftrightarrow x = -\frac{3\pi}{4} \lor x = -\cancel{\frac{5\pi}{4}}$

$k = 0: \quad x = \frac{\pi}{4} \lor x = -\frac{\pi}{4}$

$k = 1: \quad x = \frac{\pi}{4} + \frac{4\pi}{4} \lor x = -\frac{\pi}{4} + \frac{4\pi}{4}$

$\Leftrightarrow x = \cancel{\frac{5\pi}{4}} \lor x = \frac{3\pi}{4}$

As soluções são $-\frac{3\pi}{4}$, $-\frac{\pi}{4}$, $\frac{\pi}{4}$ e $\frac{3\pi}{4}$.

4.2 $f(x) = 2 \Leftrightarrow 2\cos(2x) = 2 \Leftrightarrow \cos(2x) = 1$

$\Leftrightarrow \cos(2x) = \cos 0 \Leftrightarrow 2x = 2k\pi\,,\ k \in \mathbb{Z}$

$\Leftrightarrow x = k\pi\,,\ k \in \mathbb{Z}$

$x \in [-\pi, \pi]$

$k = 1 \longrightarrow x = \pi$

$k = 0 \longrightarrow x = 0$

$k = -1 \longrightarrow x = -\pi$

As soluções são $-\pi$, 0 e π.

4.3 $f(x) = -\sqrt{2} \Leftrightarrow 2\cos(2x) = -\sqrt{2}$

$\Leftrightarrow \cos(2x) = -\frac{\sqrt{2}}{2} \Leftrightarrow \cos(2x) = \cos\frac{3\pi}{4}$

$\Leftrightarrow 2x = \frac{3\pi}{4} + 2k\pi \lor 2x = -\frac{3\pi}{4} + 2k\pi\,,\ k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{3\pi}{8} + k\pi \lor x = -\frac{3\pi}{8} + k\pi\,,\ k \in \mathbb{Z}$

Como $x \in [-\pi, \pi]$

$k = 1: \quad x = \frac{3\pi}{8} + \frac{8\pi}{8} \lor x = -\frac{3\pi}{8} + \frac{8\pi}{8}$

$\Leftrightarrow x = \cancel{\frac{11\pi}{8}} \lor x = \frac{5\pi}{8}$

$k = 0: \quad x = \frac{3\pi}{8} \lor x = -\frac{3\pi}{8}$

$k = -1: x = \frac{3\pi}{8} - \frac{8\pi}{8} \lor x = -\frac{3\pi}{8} - \frac{8\pi}{8}$

$\Leftrightarrow x = -\frac{5\pi}{8} \lor x = -\cancel{\frac{11\pi}{8}}$

As soluções são $-\frac{5\pi}{8}$, $-\frac{3\pi}{8}$, $\frac{3\pi}{8}$ e $\frac{5\pi}{8}$.

4.4 $f(x) = -0{,}2 \Leftrightarrow 2\cos(2x) = -0{,}2$

$\Leftrightarrow \cos(2x) = -0{,}1$

$\Leftrightarrow \cos(2x) = \cos(1{,}67)$ (2 c. d.)

$\Leftrightarrow 2x = 1{,}67 + 2k\pi \lor 2x = -1{,}67 + 2k\pi\,,$ $k \in \mathbb{Z}$

$\Leftrightarrow x = 0{,}84 + k\pi \lor x = -0{,}84 + k\pi\,,\ k \in \mathbb{Z}$

$k = 1: \quad x = \cancel{0{,}84} + \pi \lor x = -0{,}84 + \pi$

$x = \cancel{3{,}98} \lor x = 2{,}3$

$k = 0: \quad x = 0{,}84 \lor x = -0{,}84$

$k = -1: x = 0{,}84 - \pi \lor x = -0{,}84 - \pi$

$\Leftrightarrow x = -2{,}3 \lor x = -\cancel{3{,}98}$

As soluções são $-2{,}3$, $-0{,}84$, $0{,}84$ e $2{,}3$.

5. $[-360°, 0]$ $\quad \cos(2x) + \cos\left(\frac{x}{2}\right) = 0$

$\Leftrightarrow \cos(2x) = -\cos\left(\frac{x}{2}\right)$

$\Leftrightarrow \cos(2x) = \cos\left(180° - \frac{x}{2}\right)$

$\Leftrightarrow 2x = 180° - \frac{x}{2} + 360°\,k \lor$

$\lor\ 2x = -180° + \frac{x}{2} + 360°\,k\,,\ k \in \mathbb{Z}$

$\Leftrightarrow \frac{2x}{\underset{(2)}{1}} + \frac{x}{2} = 180° + 360°\,k \lor$

$\lor\ \frac{2x}{\underset{(2)}{1}} - \frac{x}{2} = -180° + 360°\,k\,,\ k \in \mathbb{Z}$

$\Leftrightarrow \frac{5x}{2} = 180° + 360°\,k \lor \frac{3x}{2} = -180° + 360°\,k,$ $k \in \mathbb{Z}$

$\Leftrightarrow 5x = 360° + 720°\,k \lor 3x = -360° + 720°\,k\,,$ $k \in \mathbb{Z}$

$\Leftrightarrow x = 72° + 144°\,k \lor x = -120° + 240°\,k\,,$ $k \in \mathbb{Z}$

$x \in [-360°, 0]$

$k = -2: x = 72° - 288° \lor x = -120° - 480°$

$\Leftrightarrow x = -216° \lor x = -\cancel{600°}$

$k = -3: x = 72° - 432° \lor x = -120° - 720°$

$\Leftrightarrow x = -360° \lor x = -\cancel{480°}$

$k = -1: x = 72° - 144° \lor x = -120° - 240°$

$\Leftrightarrow x = -72° \lor x = -360°$

$k = 0: \quad x = \cancel{72°} \lor x = -120°$

$k = 1: \quad x = 72° + 144° \lor x = -120° + 240°$

$\Leftrightarrow x = \cancel{216°} \lor x = 0°$

As soluções são $-360°$, $-216°$, $-120°$, $-72°$.

6. $f: x \curvearrowright y = \tan\left(\frac{x}{2}\right)$ **Pág. 225**

$x \in [-\pi, 0]$

6.1. $f(x) = 0 \Leftrightarrow \tan\frac{x}{2} = 0 \Leftrightarrow \tan\frac{x}{2} = \tan 0$

$\Leftrightarrow \frac{x}{2} = 0 + k\pi \Leftrightarrow x = 2k\pi\,,\ k \in \mathbb{Z}$

$x \in [-\pi, 0]$

$k = -1 \longrightarrow x = -\cancel{2\pi}$

$k = 0 \longrightarrow x = 0$

$k = 1 \longrightarrow x = \cancel{2\pi}$

A solução é 0.

6.2 $f(x) = \sqrt{3} \Leftrightarrow \tan\left(\frac{x}{2}\right) = \sqrt{3}$

$\Leftrightarrow \tan\frac{x}{2} = \tan\frac{\pi}{3} \Leftrightarrow \frac{x}{2} = \frac{\pi}{3} + k\pi$

$\Leftrightarrow x = \frac{2\pi}{3} + 2k\pi\,,\ k \in \mathbb{Z}$

$x \in [-\pi, 0]$

$k = -1: x = \frac{2\pi}{3} - \frac{2\pi}{\underset{(3)}{1}} \Leftrightarrow x = \frac{2\pi}{3} - \frac{6\pi}{3}$

$\Leftrightarrow x = -\cancel{\frac{4\pi}{3}}$

$k = 0: x = \cancel{\frac{2\pi}{3}}$

$k = 1: x = \frac{2\pi}{3} + 2\pi$

Não existem soluções no intervalo $[-\pi, 0]$.

6.3 $f(x)=-1 \Leftrightarrow \tan\left(\frac{x}{2}\right)=-1$

$\Leftrightarrow \tan\frac{x}{2}=\tan\left(-\frac{\pi}{4}\right) \Leftrightarrow \frac{x}{2}=-\frac{\pi}{4}+k\pi,$ $k\in\mathbb{Z}$

$\Leftrightarrow x=-\frac{\pi}{2}+2k\pi,\ k\in\mathbb{Z}$

$x\in[-\pi,\ 0]$

$k=-1: x=-\frac{\pi}{2}-\frac{2\pi}{\underset{(2)}{1}} \Leftrightarrow x=-\frac{\pi}{2}-\frac{4\pi}{2}$

$\Leftrightarrow x=\cancel{-\frac{5\pi}{2}}$

$k=0: x=-\frac{\pi}{2}$

$k=1: x=-\frac{\pi}{2}+\frac{2\pi}{\underset{(2)}{1}} \Leftrightarrow x=-\frac{\pi}{2}+\frac{4\pi}{2}$

$\Leftrightarrow x=\cancel{\frac{3\pi}{2}}$

A solução é $-\frac{\pi}{2}$.

6.4 $f(x)=-2 \Leftrightarrow \tan\left(\frac{x}{2}\right)=-2$

$\Leftrightarrow \frac{x}{2}=\tan^{-1}(-2)+k\pi,\ k\in\mathbb{Z}$

$\Leftrightarrow x=-2{,}21+2k\pi,\ k\in\mathbb{Z}$

$x\in[-\pi,\ 0]$

$k=0: x=-2{,}21$ rad

$k=1: x=-2{,}21+2\pi \Leftrightarrow x=\cancel{4{,}07}$

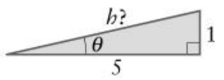
A solução é $-2{,}21$ rad.

7.1 $f(x)=a+b\sin^2 x \quad f(\theta)=?$ **Pág. 228**

$a=-1 \quad b=5$

$f(\theta)=-1+5\sin^2\theta \quad \tan\theta=\frac{1}{5}$

$\tan\theta=\frac{\text{cat. op. }\theta}{\text{cat. adj. }\theta}=\frac{1}{5}$



Cálculo auxiliar

Teorema de Pitágoras

$h^2=a^2+b^2$

$\Leftrightarrow h^2=1^2+5^2$

$\Leftrightarrow h=\cancel{\pm}\sqrt{26}$

$\sin\theta=\frac{\text{cat. op. }\theta}{\text{hip.}} \Leftrightarrow \sin\theta=\pm\frac{1}{\sqrt{26}}$

(pois não sabemos a que quadrante pertence θ)

$\sin^2\theta=\frac{1}{26}$

$f(\theta)=-1+5\times\frac{1}{26}$

$\Leftrightarrow f(\theta)=-\frac{1}{\underset{(26)}{1}}+\frac{5}{26}$

$\Leftrightarrow f(\theta)=-\frac{26}{26}+\frac{5}{26} \Leftrightarrow f(\theta)=-\frac{21}{26}$

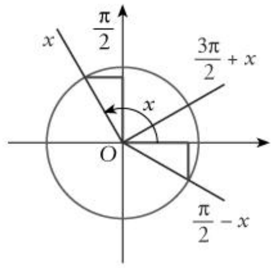
$f(\theta)=-\frac{21}{26}$

7.2 $\cos\left(\frac{7\pi}{2}+x\right)=?\quad \sin\left(\frac{\pi}{2}-x\right)=-\frac{1}{5},\ x\in 2.^\circ\ \text{Q}$

$\sin\left(\frac{\pi}{2}-x\right)=-\frac{1}{5} \Leftrightarrow \cos x=-\frac{1}{5}$

7.2 $\cos\left(\frac{7\pi}{2}+x\right)=?\quad \sin\left(\frac{\pi}{2}-x\right)=-\frac{1}{5},\ x\in 2.^\circ\ \text{Q}$

$\sin\left(\frac{\pi}{2}-x\right)=-\frac{1}{5} \Leftrightarrow \cos x=-\frac{1}{5}$



$\cos\left(\frac{7\pi}{2}+x\right)=\cos\left(\frac{4\pi}{2}+\frac{3\pi}{2}+x\right)$

$=\cos\left(\frac{3\pi}{2}+x\right)=\sin x$

Fórmula Fundamental da Trigonometria

$\sin^2 x+\cos^2 x=1 \Leftrightarrow \sin^2 x+\left(-\frac{1}{5}\right)^2=1$

$\Leftrightarrow \sin^2 x+\frac{1}{25}=1 \Leftrightarrow \sin^2 x=\frac{24}{25}$

$\Leftrightarrow \sin x=\cancel{\pm}\frac{\sqrt{24}}{5}$, porque $x\in 2.^\circ$ Q

Logo, $\cos\left(\frac{7\pi}{2}+x\right)=\sin x=\frac{\sqrt{24}}{5}=\frac{2\sqrt{6}}{5}$.

8.1 $\frac{\sin x}{\tan(-x)}=-\cos x$ **Pág. 229**

$\frac{\sin x}{\tan(-x)}=\frac{\sin x}{-\tan x}=\frac{\sin x}{-\frac{\sin x}{\cos x}}$

$=\frac{-\cancel{\sin x}\cos x}{\cancel{\sin x}}=-\cos x$

8.2 $\cos x-\cos x\sin^2 x=\cos^3 x$

$\cos x(1-\sin^2 x)=\cos x\cdot\cos^2 x=\cos^3 x$

8.3 $\frac{\sin x}{1+\cos x}=\frac{1-\cos x}{\sin x}$

$\frac{\sin^2 x}{(1+\cos x)\sin x}=\frac{\sin^2 x}{\sin x+\cos x\sin x}=$

$=\frac{1-\cos^2 x}{\sin x+\cos x\sin x}=\frac{1-\cos^2 x}{\sin x+\sin x\cos x}$

$=\frac{(1-\cos x)\cancel{(1+\cos x)}}{\sin x\cancel{(1+\cos x)}}$

$=\frac{1-\cos x}{\sin x}$

8.4 $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2+2\tan^2\theta$

$\frac{1}{\underset{(1-\sin\theta)}{1+\sin\theta}}+\frac{1}{\underset{(1-\sin\theta)}{1-\sin\theta}}=$

$=\frac{1-\sin\theta}{1^2-\sin^2\theta}+\frac{1+\sin\theta}{1^2-\sin^2\theta}=$

$=\frac{1-\cancel{\sin\theta}+1+\cancel{\sin\theta}}{1-\sin^2\theta}=\frac{2}{\cos^2\theta}=\frac{2}{\frac{1}{\tan^2\theta+1}}$

$=2(\tan^2\theta+1)=2+2\tan^2\theta$

8.5 $\frac{1}{1+\cos\theta}+\frac{1}{1-\cos\theta}=2+\frac{2}{\tan^2\theta}$

$\underset{(1-\cos\theta)}{\frac{1}{1+\cos\theta}}+\underset{(1+\cos\theta)}{\frac{1}{1-\cos\theta}}$

$=\frac{1-\cos\theta}{1^2-\cos^2\theta}+\frac{1+\cos\theta}{1^2-\cos^2\theta}$

$=\frac{1-\cancel{\cos\theta}+1+\cancel{\cos\theta}}{1-\cos^2\theta}=\frac{2}{\sin^2\theta}$

$=2\times\left(1+\frac{1}{\tan^2\theta}\right)=2+\frac{2}{\tan^2\theta}$

9.1 $\ln\left|\frac{1}{\cos x}\right|+\ln|\cos x|=\ln\left[\left|\frac{1}{\cos x}\right|\ |\cos x|\right]$

$=\ln 1=0$

9.2 a) $\frac{\sin^2 x-\cos^2 x}{\sin x+\cos x}=\frac{\sin^2 x}{\sin x}-\frac{\cos^2 x}{\cos x}$

$=\sin x-\cos x$

$\frac{\sin^2 x-\cos^2 x}{\sin x+\cos x}\neq\frac{\sin^2 x}{\sin x}-\frac{\cos^2 x}{\cos x}$

$\frac{\sin^2 x-\cos^2 x}{\sin x+\cos x}=\frac{\sin^2 x}{\sin x+\cos x}-\frac{\cos^2 x}{\sin x+\cos x}$

b) $\frac{\sin^2 x-\cos^2 x}{\sin x+\cos x}$

$=\frac{(\sin x-\cos x)(\cancel{\sin x+\cos x})}{\cancel{\sin x+\cos x}}$

$=\sin x-\cos x$

10.1 a) $\cos 105°=\cos(60°+45°)$ **Pág. 232**

$=\cos 60°\cos 45°-\sin 60°\sin 45°$

$=\frac{1}{2}\times\frac{\sqrt{2}}{2}-\frac{\sqrt{3}}{2}\times\frac{\sqrt{2}}{2}=\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}$

$=\frac{\sqrt{2}-\sqrt{6}}{4}$

$\cos 105°=\frac{\sqrt{2}-\sqrt{6}}{4}$

b) $\sin\frac{\pi}{12}=\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$

$=\sin\frac{\pi}{3}\cos\frac{\pi}{4}-\cos\frac{\pi}{3}\times\sin\frac{\pi}{4}$

$=\frac{\sqrt{3}}{2}\times\frac{\sqrt{2}}{2}-\frac{1}{2}\times\frac{\sqrt{2}}{2}=\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}$

$=\frac{\sqrt{6}-\sqrt{2}}{4}$

$\sin\frac{\pi}{12}=\frac{\sqrt{6}-\sqrt{2}}{4}$

c) $\tan\left(\frac{7\pi}{12}\right)=\tan\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$

$=\frac{\tan\frac{\pi}{3}+\tan\frac{\pi}{4}}{1-\tan\frac{\pi}{3}\times\tan\frac{\pi}{4}}=\frac{\sqrt{3}+1}{1-\sqrt{3}\times 1}$

$=\frac{\sqrt{3}+1}{-\sqrt{3}+1}=\frac{(\sqrt{3}+1)(\sqrt{3}+1)}{-3+1}$

$=\frac{4+2\sqrt{3}}{-2}=-2-\sqrt{3}$

$\tan\left(\frac{7\pi}{12}\right)=\frac{\sqrt{3}+1}{-\sqrt{3}+1}=-2-\sqrt{3}$

d) $\sin 30°=\frac{1}{2}$

10.2 $\frac{2\tan\alpha}{1+\tan^2\alpha}=\sin(2\alpha)$

$\Leftrightarrow\frac{2\tan\alpha}{1+\tan^2\alpha}=2\sin\alpha\cos\alpha$

$\frac{2\tan\alpha}{1+\tan^2\alpha}=\frac{2\tan\alpha}{1+\frac{\sin^2\alpha}{\cos^2\alpha}}=\frac{2\tan\alpha}{\frac{\cos^2\alpha+\sin^2\alpha}{\cos^2\alpha}}$

$=\frac{2\tan\alpha}{\frac{1}{\cos^2\alpha}}=2\tan\alpha\cos^2\alpha=2\frac{\sin\alpha}{\cos\alpha}\cos^2\alpha$

$=2\sin\alpha\cos\alpha$

11. $f(x)=\cos\left(x+\frac{3\pi}{2}\right)$ **Pág. 233**

$=\cos x\times\cos\frac{3\pi}{2}-\sin x\times\sin\frac{3\pi}{2}$

$=\cos x\times 0-\sin x\times(-1)=\sin x$

$f(x)=\sin x$

12.1 $d=50\sin\theta\cos\theta$, d em metros

$\Leftrightarrow\ d=25\times 2\sin\theta\cos\theta$

$\Leftrightarrow\ d=25\sin(2\theta)$

12.2 $d=25\sin(2\theta)$

$0<\theta<\frac{\pi}{2}$

$\Leftrightarrow\ 0<2\theta<\pi$

$\Leftrightarrow\ \alpha\sin(2\theta)\leqslant 1$

$\Leftrightarrow\ 0<\underbrace{25\sin(2\theta)}_{d}\leqslant 25$

d é máxima quando $d=25$, ou seja:

$25\sin(2\theta)=25\ \Leftrightarrow\ \sin(2\theta)=1$

$\Leftrightarrow\ \sin(2\theta)=\sin\frac{\pi}{2}$

$\Leftrightarrow\ 2\theta=\frac{\pi}{2}+2k\pi,\ k\in\mathbb{Z}$

$\Leftrightarrow\ \theta=\frac{\pi}{4}+k\pi,\ k\in\mathbb{Z}$

Como $\theta\in\left]0,\frac{\pi}{2}\right[$, então $\theta=\frac{\pi}{4}$

d é máximo quando $\theta=\frac{\pi}{4}$ rad .

13.1 $\sin\left(\frac{\pi}{8}\right)=\sin\left(\frac{\pi/4}{2}\right)=\sqrt{\frac{1-\cos\left(\frac{\pi}{4}\right)}{2}}=$ **Pág. 234**

$=\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}=\sqrt{\frac{2-\sqrt{2}}{4}}=\frac{\sqrt{2-\sqrt{2}}}{2}$

13.2 $\cos\left(\frac{\pi}{8}\right)=\cos\left(\frac{\pi/4}{2}\right)=\sqrt{\frac{1+\cos\frac{\pi}{4}}{2}}=$

$=\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}=\sqrt{\frac{2+\sqrt{2}}{4}}=\frac{\sqrt{2+\sqrt{2}}}{2}$

Alternativamente, pode-se determinar $\cos\left(\frac{\pi}{8}\right)$ a partir do resultado obtido na alínea anterior. Assim, recorrendo à fórmula fundamental da trigonometria tem-se:

$\sin^2\left(\frac{\pi}{8}\right)+\cos^2\left(\frac{\pi}{8}\right)=1$

Como $\sin\frac{\pi}{8}=\frac{\sqrt{2-\sqrt{2}}}{2}$, vem:

$$\left(\frac{\sqrt{2-\sqrt{2}}}{2}\right)^2+\cos^2\left(\frac{\pi}{8}\right)=1$$

$$\Leftrightarrow \cos^2\left(\frac{\pi}{8}\right)=1-\frac{2-\sqrt{2}}{2} \Leftrightarrow \cos^2\left(\frac{\pi}{8}\right)=\frac{4-2+\sqrt{2}}{4}$$

$$\Leftrightarrow \cos^2\left(\frac{\pi}{8}\right)=\frac{2+\sqrt{2}}{4} \Leftrightarrow \cos\left(\frac{\pi}{8}\right)=\frac{\sqrt{2+\sqrt{2}}}{2}$$

13.3 $\tan\left(\frac{\pi}{8}\right)=\tan\left(\frac{\pi/4}{2}\right)=\sqrt{\frac{1-\cos\left(\frac{\pi}{4}\right)}{1+\cos\left(\frac{\pi}{4}\right)}}=$

$$=\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}}=\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}}=\sqrt{\frac{\left(2-\sqrt{2}\right)\left(2-\sqrt{2}\right)}{\left(2+\sqrt{2}\right)\left(2-\sqrt{2}\right)}}=$$

$$=\sqrt{\frac{\left(2-\sqrt{2}\right)^2}{2^2-\left(\sqrt{2}\right)^2}}=\frac{2-\sqrt{2}}{2}$$

14.1 $\sin\frac{x}{2}=\pm\sqrt{\frac{1-\cos x}{2}}$

Considerando que:

$\sin^2 x+\cos^2 x=1$ e $\sin x=-\frac{\sqrt{3}}{2}$, vem:

$$\left(-\frac{\sqrt{3}}{2}\right)^2+\cos^2 x=1 \Leftrightarrow \cos^2 x=\frac{1}{4} \Leftrightarrow \cos x=\pm\frac{1}{2}$$

Uma vez que $\frac{\pi}{2}<x<\frac{3\pi}{2}$, então $\cos x=-\frac{1}{2}$.

Assim, tem-se que:

$$\sin\frac{x}{2}=\pm\sqrt{\frac{1-\left(-\frac{1}{2}\right)}{2}}=\pm\sqrt{\frac{3}{4}}=\pm\frac{\sqrt{3}}{2}$$

Como $\frac{\pi}{4}<\frac{x}{2}<\frac{3\pi}{4}$, a solução é $\frac{\sqrt{3}}{2}$.

$\cos\frac{x}{2}=\pm\sqrt{\frac{1+\cos x}{2}}$

Considerando que:

$\sin^2 x+\cos^2 x=1$ e $\sin x=-\frac{\sqrt{3}}{2}$, vem:

$$\left(-\frac{\sqrt{3}}{2}\right)^2+\cos^2 x=1 \Leftrightarrow \cos^2 x=\frac{1}{4} \Leftrightarrow \cos x=\pm\frac{1}{2}$$

Uma vez que $\frac{\pi}{2}<x<\frac{3\pi}{2}$, então $\cos x=-\frac{1}{2}$.

Assim, tem-se que:

$$\cos\frac{x}{2}=\pm\sqrt{\frac{1+\left(-\frac{1}{2}\right)}{2}}=\pm\sqrt{\frac{1}{4}}=\pm\frac{1}{2}$$

Como $\frac{\pi}{4}<\frac{x}{2}<\frac{3\pi}{4}$, as soluções são $-\frac{1}{2}$ e $\frac{1}{2}$.

Nota: Por lapso, a solução que consta do manual não está correcta.

15.1 $\sin(2x)\times\cos(3x)=$ **Pág. 236**

$$=\sin\left(\frac{5x-x}{2}\right)\times\cos\left(\frac{5x+x}{2}\right)=$$

$$=\frac{1}{2}\left[\sin(5x)-\text{sen}(x)\right]=\frac{1}{2}\left[\sin(5x)+\text{sen}(-x)\right]=$$

$$=\frac{1}{2}\sin(5x)+\frac{1}{2}\sin(-x)$$

15.2 $\cos(2x)\times\cos(5x)=$

$$=\cos\left(\frac{7x-3x}{2}\right)\times\cos\left(\frac{7x+3x}{2}\right)=$$

$$=\frac{1}{2}\left[\cos(7x)+\cos(3x)\right]=$$

$$=\frac{1}{2}\cos(7x)+\frac{1}{2}\cos(3x)$$

---

16. $\sin(3x)+\sin(6x)=0$ **Pág. 237**

$$\Leftrightarrow \sin\left(\frac{9x}{2}\right)\times\cos\left(\frac{-3x}{2}\right)=0$$

$$\Leftrightarrow \sin\left(\frac{9x}{2}\right)=0 \vee \cos\left(\frac{3x}{2}\right)=0, \text{ pois } \cos(-\alpha)=\cos(\alpha)$$

$$\Leftrightarrow \frac{9x}{2}=k\pi \vee \frac{3x}{2}=\frac{\pi}{2}+k\pi,\ k\in\mathbb{Z}$$

$$\Leftrightarrow x=\frac{2}{9}k\pi \vee x=\frac{\pi}{3}+\frac{2k\pi}{3},\ k\in\mathbb{Z}$$

17.1 $\cos x+\cos(2x)=0$

$$\Leftrightarrow 2\cos\left(\frac{3x}{2}\right)\times\cos\left(\frac{-x}{2}\right)$$

$$\Leftrightarrow \cos\left(\frac{3x}{2}\right)=0 \vee \cos\left(-\frac{x}{2}\right)=0$$

$$\Leftrightarrow \frac{3x}{2}=\frac{\pi}{2}+k\pi \vee -\frac{x}{2}=\frac{\pi}{2}+k\pi,\ k\in\mathbb{Z}$$

$$\Leftrightarrow x=\frac{\pi}{3}+\frac{2k\pi}{3} \vee x=-\pi-2k\pi,\ k\in\mathbb{Z}$$

Então, se:

$k=-1 \to x=-\frac{\pi}{3} \vee x=-3\pi$

$k=0 \to x=\frac{\pi}{3} \vee x=-\pi$

$k=1 \to x=\pi \vee x=-\frac{3}{2}\pi$

Logo, no intervalo $[-\pi,\pi]$, as soluções são:

$-\pi,\ -\frac{\pi}{3},\ \frac{\pi}{3}$ e π.

17.2 $\cos x+\text{sen}(2x)=0$

$$\Leftrightarrow \cos x+\cos\left(\frac{\pi}{2}-2x\right)=0$$

$$\Leftrightarrow 2\cos\left(\frac{-x+\pi/2}{2}\right)\times\cos\left(\frac{3x-\pi/2}{2}\right)=0$$

$$\Leftrightarrow \cos\left(-\frac{x}{2}+\frac{\pi}{4}\right)=0 \vee \cos\left(\frac{3x}{2}-\frac{\pi}{4}\right)=0$$

$$\Leftrightarrow -\frac{x}{2}+\frac{\pi}{4}=\frac{\pi}{2}+k\pi \vee \frac{3x}{2}-\frac{\pi}{4}=\frac{\pi}{2}+k\pi,\ k\in\mathbb{Z}$$

$$\Leftrightarrow x=-\frac{\pi}{2}-2k\pi \vee x=\frac{\pi}{2}+\frac{2k\pi}{3},\ k\in\mathbb{Z}$$

Então, se:

$k=-2 \to x=\frac{7\pi}{2} \vee x=-\frac{5\pi}{6}$

$k=-1 \to x=\frac{3\pi}{2} \vee x=-\frac{\pi}{6}$

$k=0 \to x=-\frac{\pi}{2} \vee x=\frac{\pi}{2}$

$k=1 \to x=-\frac{5\pi}{2} \vee x=\frac{7\pi}{6}$

Logo, no intervalo $[-\pi,\pi]$, as soluções são:

$-\frac{5\pi}{6},\ -\frac{\pi}{2},\ -\frac{\pi}{6}$ e $\frac{\pi}{2}$.

18.1 $\sin x + \sin\left(\frac{\pi}{3}+x\right) > \frac{3}{2}$

$\Leftrightarrow 2\sin\left(\frac{2x+\pi/3}{2}\right)\times\cos\left(\frac{-\pi/3}{2}\right) > \frac{3}{2}$

$\Leftrightarrow 2\sin\left(x+\frac{\pi}{6}\right)\times\cos\left(-\frac{\pi}{6}\right) > \frac{3}{2}$

$\Leftrightarrow 2\sin\left(x+\frac{\pi}{6}\right)\times\frac{\sqrt{3}}{2} > \frac{3}{2}$

$\Leftrightarrow \sin\left(x+\frac{\pi}{6}\right) > \frac{3}{2\sqrt{3}} \Leftrightarrow \sin\left(x+\frac{\pi}{6}\right) > \frac{\sqrt{3}}{2}$

$\Leftrightarrow \frac{\pi}{3}+2k\pi < x+\frac{\pi}{6} < \frac{2\pi}{3}+2k\pi,\ k\in\mathbb{Z}$

$\Leftrightarrow \frac{\pi}{6}+2k\pi < x < \frac{\pi}{2}+2k\pi,\ k\in\mathbb{Z}$

Então, se:

$k=-1 \rightarrow -\frac{11}{6}\pi < x < -\frac{3}{2}\pi$

$k=0 \rightarrow \frac{\pi}{6} < x < \frac{\pi}{2}$

$k=1 \rightarrow \frac{13}{6}\pi < x < \frac{5}{2}\pi$

Logo, no intervalo $[-\pi, \pi]$, tem-se que:

$S = \left]\frac{\pi}{6}, \frac{\pi}{2}\right[$

Nota: Por lapso, a solução apresentada no manual não está correcta.

18.2 $|\sin x - \cos x| > 1$

$\Leftrightarrow \sin x - \cos x > 1 \vee \sin x - \cos x < -1$

$\Leftrightarrow \sin x - \sin\left(\frac{\pi}{2}+x\right) > 1 \vee \sin x - \sin\left(\frac{\pi}{2}+x\right) < -1$

$\Leftrightarrow 2\sin\left(\frac{-\pi/2}{2}\right)\times\cos\left(\frac{2x+\pi/2}{2}\right) > 1 \vee$

$\vee\ 2\sin\left(\frac{-\pi/2}{2}\right)\times\cos\left(\frac{2x+\pi/2}{2}\right) < -1$

$\Leftrightarrow 2\sin\left(-\frac{\pi}{4}\right)\times\cos\left(x+\frac{\pi}{4}\right) > 1 \vee$

$\vee\ 2\sin\left(-\frac{\pi}{4}\right)\times\cos\left(x+\frac{\pi}{4}\right) < -1$

$\Leftrightarrow -\sqrt{2}\cos\left(x+\frac{\pi}{4}\right) > 1 \vee -\sqrt{2}\cos\left(x+\frac{\pi}{4}\right) < -1$

$\Leftrightarrow \cos\left(x+\frac{\pi}{4}\right) < -\frac{\sqrt{2}}{2} \vee \cos\left(x+\frac{\pi}{4}\right) > \frac{\sqrt{2}}{2}$

$\Leftrightarrow \frac{3\pi}{4}+2k\pi < x+\frac{\pi}{4} < \frac{5\pi}{4}+2k\pi \vee$

$\vee -\frac{\pi}{4}+2k\pi < x+\frac{\pi}{4} < \frac{\pi}{4}+2k\pi,\ k\in\mathbb{Z}$

$\Leftrightarrow \frac{\pi}{2}+2k\pi < x < \pi+2k\pi \vee$

$\vee -\frac{\pi}{2}+2k\pi < x < 0+2k\pi,\ k\in\mathbb{Z}$

Então, se:

$k=-1 \rightarrow -\frac{3}{2}\pi < x < -\pi \vee -\frac{5}{2}\pi < x < -2\pi$

$k=0 \rightarrow \frac{\pi}{2} < x < \pi \vee -\frac{\pi}{2} < x < 0$

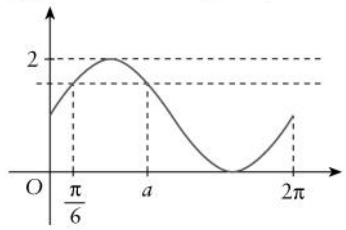
$k=1 \rightarrow \frac{5}{2}\pi < x < 5\pi \vee \frac{3}{2}\pi < x < 2\pi$

Logo, no intervalo $[-\pi, \pi]$, tem-se que:

$S = \left]-\frac{\pi}{2}, 0\right[\cup \left]\frac{\pi}{2}, \pi\right[$

Pág. 238

1. $f(x) = \sin x + 1,\ x \in [0, 2\pi]$



$f(a) = f\left(\frac{\pi}{6}\right) \wedge a \in [0, 2\pi]$

$\sin a + 1 = \sin\frac{\pi}{6} + 1 \wedge a \in [0, 2\pi]$

$a = \frac{\pi}{6} \vee a = \pi - \frac{\pi}{6}$

$a = \frac{\pi}{6} \vee \frac{5\pi}{6}$

(D) .

2. $\cos(\alpha x) = 0$

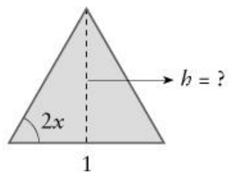
$\Leftrightarrow \alpha x = \frac{\pi}{2}+2k\pi \vee \alpha x = \frac{3}{2}\pi+2k\pi,\ k\in\mathbb{Z}$

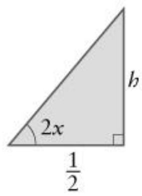
(B)

3. $\sin(2x) = 0 \Longleftrightarrow \frac{\sin(2x)}{2} = 0$

(C) .

4. Triângulo isósceles de base 1





$\tan a = \frac{\text{cat. op. a } a}{\text{cat. adj. a } a}$

$\tan(2x) = \frac{h}{\frac{1}{2}} \Longleftrightarrow h = 0{,}5\tan(2x)$

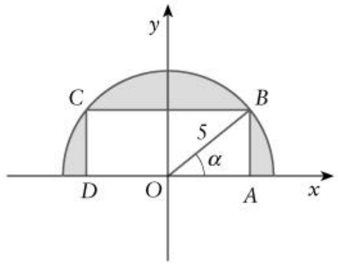
$h = 0{,}5\tan(2x)$

$A_{\triangle} = \frac{b\times h}{2} = \frac{1\times 0{,}5\tan(2x)}{2}$

$= \frac{\tan(2x)}{4}$

(C) .

5. Rectângulo $[ABCD]$ inscrito na semicircunferência de centro O e raio 5 cm .



$A_{\text{Parte colorida}} = A_{◓} - A_{▬}$ (2)

$A_{◓} = \frac{A_{\odot}}{2} = \frac{\pi r^2}{2} = \frac{25\pi}{2}$

$A_{◓} = \frac{25\pi}{2}$

$A_{\blacksquare} = 2 \times \overline{OA} \times \overline{AB}$ (1)

$\cos x = \frac{\text{cat. adj. a } x}{\text{hip.}} \Leftrightarrow \cos x = \frac{\overline{OA}}{5}$

$\Leftrightarrow \overline{OA} = 5 \cos x$

$\sin x = \frac{\text{cat. op. a } x}{\text{hip.}} \Leftrightarrow \sin x = \frac{\overline{AB}}{5}$

$\Leftrightarrow \overline{AB} = 5 \sin x$

Voltando a (1) vem:

$A_{\blacksquare} = 2 \times \overline{OA} \times \overline{AB} = 2 \times 5 \cos x \times 5 \sin x$

$= 50 \cos x \sin x$

$\therefore A_{\blacksquare} = 50 \sin x \cos x$

Em (2) $A_{\text{Parte colorida}} = A_{\frown} - A_{\blacksquare}$

$= \frac{25\pi}{2} - 50 \sin x \cos x = \frac{25\pi}{2} - 25 \times \underbrace{2 \sin x \cos x}_{\sin (2x)}$

$= \frac{25\pi}{2} - 25 \sin (2x)$

(A) .

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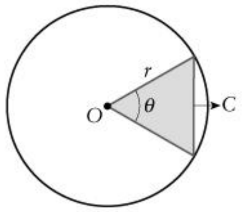
6. **Pág. 239**



$C = 2r \sin \frac{\theta}{2}$

6.1. $c = \sqrt{2} r$

$c = 2r \sin \frac{\theta}{2} \Leftrightarrow \sqrt{2} r = 2r \sin \frac{\theta}{2}$

$\Leftrightarrow \sin \frac{\theta}{2} = \frac{\sqrt{2}}{2}$

$\Leftrightarrow \sin \frac{\theta}{2} = \sin \frac{\pi}{4}$

$\Leftrightarrow \underset{(2)}{\frac{\theta}{2}} = \frac{\pi}{4} + \underset{(4)}{\frac{2k\pi}{1}} \vee \frac{\theta}{2} = \left(\pi - \frac{\pi}{4}\right) + 2k\pi$,

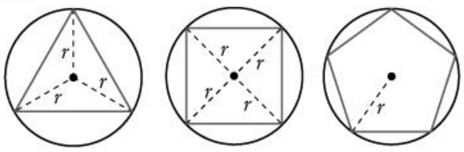
$k \in \mathbb{Z}$

$\Leftrightarrow 2\theta = \pi + 8k\pi \vee \underset{(2)}{\frac{\theta}{2}} = \frac{3\pi}{4} + \underset{(4)}{\frac{2k\pi}{1}}$,
$k \in \mathbb{Z}$

$\Leftrightarrow \theta = \frac{\pi}{2} + 4k\pi \vee 2\theta = 3\pi + 8k\pi$, $k \in \mathbb{Z}$

$\Leftrightarrow \theta = \frac{\pi}{2} + 4k\pi \vee \theta = \frac{3\pi}{2} + 4k\pi$, $k \in \mathbb{Z}$

6.2



3 lados: $l = r\sqrt{3}$; 4 lados: $l = r\sqrt{2}$;
6 lados: $l = r$.

7.1 $\frac{1}{\sin x} = \frac{1}{2} \Leftrightarrow$

$\Leftrightarrow \sin x = 2 \longrightarrow$ impossível

porque $-1 \leqslant \sin x \leqslant 1$.

8.1 $\tan x \cos x = \tan x$

$\Leftrightarrow \tan x \cos x - \tan x = 0$

$\Leftrightarrow \tan x (\cos x - 1) = 0$

$\Leftrightarrow \tan x = 0 \vee \cos x = 1$

$\Leftrightarrow \tan x = \tan 0 \vee \cos x = \cos 0$

$\Leftrightarrow x = 0 + k\pi \vee x = 0 + 2k\pi$, $k \in \mathbb{Z}$

$\Leftrightarrow x = k\pi \vee x = 2k\pi$, $k \in \mathbb{Z}$

$\Leftrightarrow x = k\pi$, $k \in \mathbb{Z}$

como $x \in [0, 2\pi]$

$k = 0: \quad x = 0$

$k = 1: \quad x = \pi$

$k = 2: \quad x = 2\pi$

As soluções da equação são 0 , π e 2π .

8.2. $3 \tan^2 x = 1 \Leftrightarrow \tan^2 x = \frac{1}{3}$

$\Leftrightarrow \tan x = \sqrt{\frac{1}{3}} \vee \tan x = -\sqrt{\frac{1}{3}}$

$\Leftrightarrow \tan x = \frac{\sqrt{3}}{3} \vee \tan x = -\frac{\sqrt{3}}{3}$

$\Leftrightarrow \tan x = \tan \frac{\pi}{6} \vee \tan x = \tan \left(\frac{11\pi}{6}\right)$

$\Leftrightarrow x = \frac{\pi}{6} + k\pi \vee x = \frac{11\pi}{6} + k\pi$, $k \in \mathbb{Z}$

como $x \in [0, 2\pi]$

$x = \frac{\pi}{6} \vee x = \frac{5\pi}{6} \vee x = \frac{7\pi}{6} \vee x = \frac{11\pi}{6}$

As soluções da equação são $\frac{\pi}{6}$, $\frac{5\pi}{6}$, $\frac{7\pi}{6}$ e $\frac{11\pi}{6}$.

9. $e(t) = 1{,}2 \sin t$, t em segundos e e em volts.

$t = ?$

$0 \leqslant t \leqslant 5$: $e(t) = 0{,}6$ volts

$e(t) = 1{,}2 \sin t \Leftrightarrow 0{,}6 = 1{,}2 \sin t$

$\Leftrightarrow \sin t = \frac{0{,}6}{1{,}2} \Leftrightarrow \sin t = \frac{1}{2}$

$\Leftrightarrow \sin t = \sin \frac{\pi}{6}$

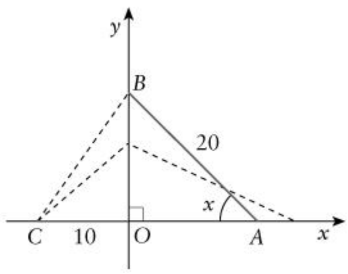
$\Leftrightarrow t = \frac{\pi}{6} + 2k\pi \vee t = \frac{5\pi}{6} + 2k\pi$, $k \in \mathbb{Z}$

Como $0 \leqslant t \leqslant 5$, $t = \frac{\pi}{6} \vee t = \frac{5\pi}{6}$

$t = 0{,}525 \vee t = 2{,}625$

10. $\overline{CO} = 10$ e $\overline{AB} = 20$;

$x = B\hat{A}O$, $x \in \left]0, \frac{\pi}{2}\right[$.



10.1 $A_{\triangle[ABC]} = f(x) = 100 (\sin x + \sin (2x))$

$A_{\triangle[ABC]} = \frac{b \times h}{2} = \frac{(10 + \overline{OA}) \times \overline{OB}}{2}$

$A_{\triangle[ABC]} = f(x) = 100\ (\sin x + \sin (2x))$

$A_{\triangle[ABC]} = \dfrac{b \times h}{2} = \dfrac{(10 + \overline{OA}) \times \overline{OB}}{2}$

$\overline{OA} = ?\quad \cos x = \dfrac{\text{cat. adj. a } x}{\text{hip.}} \Leftrightarrow \cos x = \dfrac{\overline{OA}}{20}$

$\Leftrightarrow \overline{OA} = 20 \cos x$

$\overline{OB} = ?\quad \sin x = \dfrac{\text{cat. op. a } x}{\text{hip.}} \Leftrightarrow \sin x = \dfrac{\overline{OB}}{20}$

$\Leftrightarrow \overline{OB} = 20 \sin x$

$A_{\triangle[ABC]} = \dfrac{(10 + \overline{OA}) \times \overline{OB}}{2}$

$\Leftrightarrow A_{\triangle[ABC]} = \dfrac{(10 + 20\cos x) \times 20 \sin x}{2}$

$\Leftrightarrow A_{\triangle[ABC]} = 10\ (10 + 20 \cos x) \sin x$

$\Leftrightarrow A_{\triangle[ABC]} = 100 \sin x + 200 \cos x \sin x$

$\Leftrightarrow A_{\triangle[ABC]} = 100\ (\sin x + 2 \sin x \cos x)$

$\Leftrightarrow A_{\triangle[ABC]} = 100\ [\sin x + \sin (2x)]$

10.2 $\sin x + \sin (2x) = 0$

$\Leftrightarrow \sin x + 2 \sin x \cos x = 0$

$\Leftrightarrow \sin x\ (1 + 2 \cos x) = 0$

$\Leftrightarrow \sin x = 0 \ \vee\ 1 + 2\cos x = 0$

$\Leftrightarrow \sin x = \sin 0 \ \vee\ \cos x = -\dfrac{1}{2}$

$\Leftrightarrow \sin x = \sin 0 \ \vee\ \cos x = \cos \dfrac{2\pi}{3}$

$\Leftrightarrow x = 2k\pi \ \vee\ x = \pi + 2k\pi \ \vee$

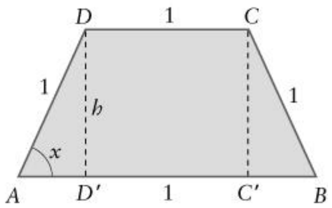
$\vee\ x = \dfrac{2\pi}{3} + 2k\pi \ \vee\ x = -\dfrac{2\pi}{3} + 2k\pi\ ,\ k \in \mathbb{Z}$

Como $x \in [0\ ,\ 2\pi]$

$x = 0 \ \vee\ x = \pi \ \vee\ x = \dfrac{2\pi}{3} \ \vee\ x = \dfrac{4\pi}{3}$

As soluções da equação são 0 , $\dfrac{2\pi}{3}$, π ou $\dfrac{4\pi}{3}$.

11.

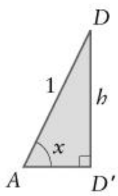


11.1 $A_{\text{Trapézio}} = A(x) = \dfrac{1}{2} \sin (2x) + \sin x$

$A_{\text{Trapézio}} = \dfrac{(B + b) \times h}{2} \Leftrightarrow$

$\Leftrightarrow A_{\text{Trapézio}} = (\overline{AB} + 1)\ \dfrac{h}{2}\quad (1)$

$h = ?\quad \sin x = \dfrac{\text{cat. op. } x}{\text{hip.}}$



$\Leftrightarrow \sin x = \dfrac{h}{1}$

$\Leftrightarrow h = \sin x$

$\cos x = \dfrac{\text{cat. adj. a } x}{\text{hip.}} \Leftrightarrow$

$\Leftrightarrow \cos x = \dfrac{\overline{AD'}}{1} \Leftrightarrow \overline{AD'} = \cos x$

Logo,

$\overline{AB} = 1 + 2\ \overline{AD'} \Leftrightarrow \overline{AB} = 1 + 2 \cos x$

Voltando a (1) vem:

$A_{\text{Trapézio}} = \dfrac{(\overline{AB} + 1)\ h}{2}$

$\Leftrightarrow A_{\text{Trapézio}} = \dfrac{(1 + 2\cos x + 1) \sin x}{2}$

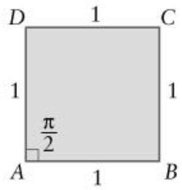
$\Leftrightarrow A_{\text{Trapézio}} = \dfrac{(2 + 2\cos x) \sin x}{2}$

$\Leftrightarrow A_{\text{Trapézio}} = \dfrac{1}{2}\ (2 \sin x + 2 \cos x \sin x)$

$\Leftrightarrow A_{\text{Trapézio}} = \dfrac{1}{2}\ [2 \sin x + \sin (2x)]$

$\Leftrightarrow A_{\text{Trapézio}}(x) = \dfrac{1}{2} \sin (2x) + \sin x$

11.2 $A\ (x) = \dfrac{1}{2} \sin (2x) + \sin x$



$A\left(\dfrac{\pi}{2}\right) = \dfrac{1}{2} \sin \left(2 \times \dfrac{\pi}{2}\right) + \sin \dfrac{\pi}{2}$

$\Leftrightarrow A\left(\dfrac{\pi}{2}\right) = \dfrac{1}{2} \sin \pi + \sin \dfrac{\pi}{2}$

$\Leftrightarrow A\left(\dfrac{\pi}{2}\right) = \dfrac{1}{2} \times 0 + 1 \Leftrightarrow A\left(\dfrac{\pi}{2}\right) = 1$ u. a.

Este resultado, significa que o "trapézio", fica transformado num quadrado de lado 1 .

12. $\sin^2 \theta = \dfrac{1 - \cos (2\theta)}{2}$

$2.^{\circ}\ \text{Membro} = \dfrac{1 - \cos (2\theta)}{2}$

$= \dfrac{1 - (\cos^2 \theta - \sin^2 \theta)}{2}$

$= \dfrac{\overbrace{1 - \cos^2 \theta}^{\sin^2 \theta} + \sin^2 \theta}{2} = \dfrac{\sin^2 \theta + \sin^2 \theta}{2}$

$= \dfrac{2 \sin^2 \theta}{2} = \sin^2 \theta = 1.^{\circ}$ membro

13. $\cos x + \text{sen}(2x) = 0$

$\Leftrightarrow \cos x + \cos\left(\dfrac{\pi}{2} - 2x\right) = 0$

$\Leftrightarrow 2\cos\left(\dfrac{-x + \pi/2}{2}\right) \times \cos\left(\dfrac{3x - \pi/2}{2}\right) = 0$

$\Leftrightarrow \cos\left(-\dfrac{x}{2} + \dfrac{\pi}{4}\right) = 0 \vee \cos\left(\dfrac{3x}{2} - \dfrac{\pi}{4}\right) = 0$

$\Leftrightarrow -\dfrac{x}{2} + \dfrac{\pi}{4} = \dfrac{\pi}{2} + k\pi \vee \dfrac{3x}{2} - \dfrac{\pi}{4} = \dfrac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$

$\Leftrightarrow x = -\dfrac{\pi}{2} - 2k\pi \vee x = \dfrac{\pi}{2} + \dfrac{2k\pi}{3},\ k \in \mathbb{Z}$

Então, se:

$k = -2 \to x = \dfrac{7\pi}{2} \vee x = -\dfrac{5\pi}{6}$

$k = -1 \to x = \dfrac{3\pi}{2} \vee x = -\dfrac{\pi}{6}$

$k = 0 \to x = -\dfrac{\pi}{2} \vee x = \dfrac{\pi}{2}$

$k=1 \rightarrow x=-\frac{5\pi}{2} \vee x=\frac{7\pi}{6}$

Logo, no intervalo $[-\pi, \pi]$, as soluções são:

$-\frac{5\pi}{6}, -\frac{\pi}{2}, -\frac{\pi}{6}$ e $\frac{\pi}{2}$.