1. Ana andou 8 voltas completas + 210°

1.1 Ângulo? (graus e radianos)

Graus

8 voltas completas = $8 \times 360° = 2880°$

$2880° + 210° = 3090°$

Radianos

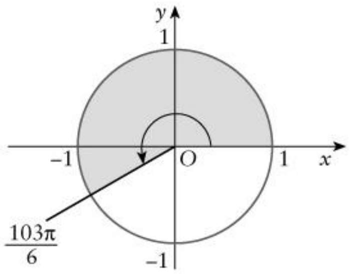
$8 \times 2\pi = 16\pi$

$210° = 180° + 30° = \underset{(6)}{\frac{\pi}{1}} + \frac{\pi}{6} = \frac{7\pi}{6}$

$\underset{(6)}{\frac{16\pi}{1}} + \frac{7\pi}{6} = \frac{96\pi}{6} + \frac{7\pi}{6} = \frac{103\pi}{6}$

R.: A Ana descreveu um ângulo de 30 $\frac{103\pi}{6}$ radianos.

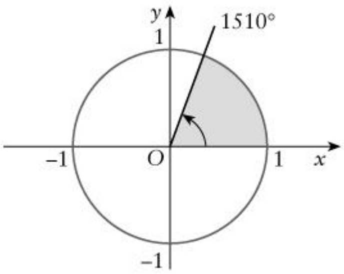
1.2



O ângulo de $\frac{103\pi}{6}$ radianos pertence

Quadrante.

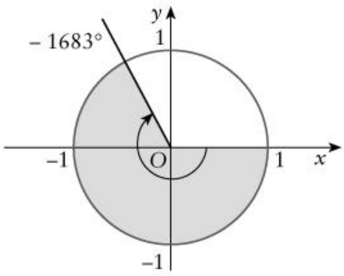
2.1 $1510° = 4 \times 360° + 70°$



2.2 − 1683°

$-1683° = 4 \times (-360°) - 243°$

$-243° = -180° - 63°$



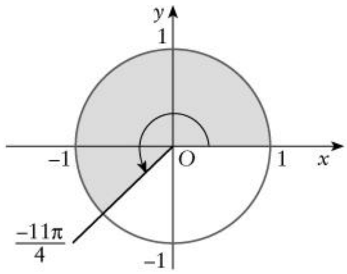
− 1683° pertence ao 2.° Quadrante.

2.3 $-\frac{11\pi}{4}$ rad

$\frac{16\pi}{4} = 2$ voltas completas

$-\frac{11\pi}{4} + \frac{16\pi}{4} = \frac{5\pi}{4}$

Logo, $-\frac{11\pi}{4} = \frac{5\pi}{4}$.



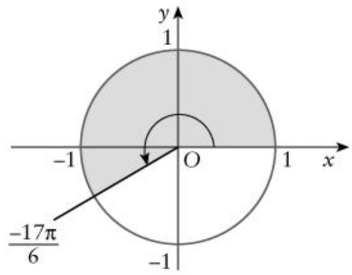
Portanto, $-\frac{11\pi}{4}$ pertence ao 3.° Quadrante.

2.4 $-\frac{17\pi}{6}$ rad

$\frac{24\pi}{6} = 4\pi = 2$ voltas completas

$-\frac{17\pi}{6} + \frac{24\pi}{6} = \frac{7\pi}{6}$

Logo, $-\frac{17\pi}{6} = \frac{7\pi}{6}$.



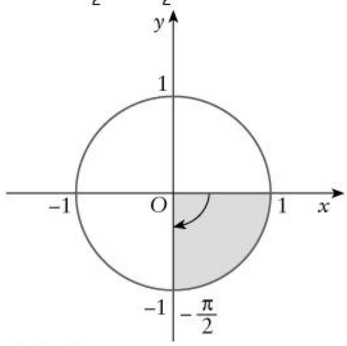
Portanto, $-\frac{17\pi}{6}$ pertence ao 3.° Quadrante.

2.5 $-\frac{100\pi}{8}$ rad $= -\frac{25\pi}{2}$ rad

$\frac{24\pi}{2} = 12\pi = 6$ voltas completas

$-\frac{25\pi}{2} + \frac{24\pi}{2} = -\frac{\pi}{2}$

Logo, $-\frac{25\pi}{2} = -\frac{\pi}{2}$.



2.6 − 8,7 rad

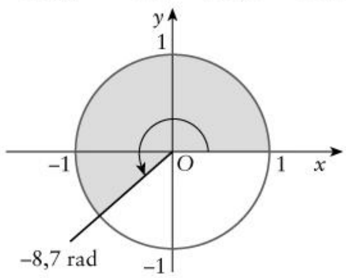
Em graus: π rad — 180°

− 8,7 rad — x

$x = 180° \times \frac{(-8{,}7 \text{ rad})}{\pi \text{ rad}} \Leftrightarrow x = \frac{-1566°}{\pi}$

$\Leftrightarrow x \approx -498{,}73°$ (2 c. d.)

$-498{,}73° = -360° - 138{,}73° = 221{,}27°$



2.7 -30 rad

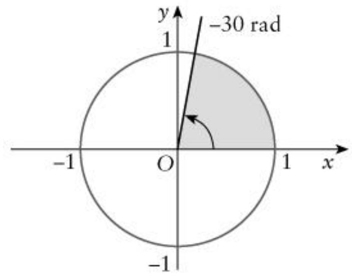
Em graus: π rad —— $180°$

-30 rad — $x°$

$x = \frac{-30\ \cancel{\text{rad}} \times 180°}{\pi\ \cancel{\text{rad}}} \Leftrightarrow x = -\frac{5400°}{\pi}$

$\Leftrightarrow x = -1720°$ (0 c.d.)

$-1720 = \underbrace{4 \times (-360°)}_{-1440} - 280° = -280° = 80°$

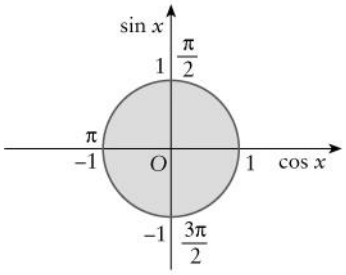


3.

Jogador	Ângulo	Prémio euros
A	$-755°$	10
B	$\frac{16\pi}{3}$ rad	50
C	-50	1000
D	30	100
E	$60° + k \times 360°$ $k \in \{1,\ 2,\ 3\}$	50

4.1 $\sin\frac{\pi}{2} - \cos\frac{3\pi}{2} + 2\cos\pi - 2\sin\frac{3\pi}{2}$ **Pág. 194**

$= 1 - 0 + 2 \times (-1) - 2 \times (-1) = 1 - 2 + 2 =$



4.2 $\sin^2\frac{\pi}{2} + \cos^2\pi + \tan\pi - 2\cos 0$

$= 1^2 + (-1)^2 + 0 - 2 \times 1 = 1 + 1 - 2 = 0$

4.3 $\sin^2\frac{3\pi}{2} + \cos^2\frac{3\pi}{2} - 1 = (-1)^2 + 0^2 - 1$

$= 1 + 0 - 1 = 0$

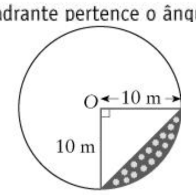
4.4 $\cos 540° - \tan 720° + \sin 990°$

$= \cos(360° + 180°) - \tan 0° + \sin(\underbrace{720°}_{2 \times 360°} + 270°)$

$= \cos 180° - \tan 0° + \sin 270°$

$= -1 - 0 + (-1) = -1 - 1 = -2$

5. A que quadrante pertence o ângulo? **Pág. 195**



5.1 $\sin\alpha = -\frac{1}{5}$ α pertence ao 3.° ou 4.° Q

5.2 $\cos\alpha = \frac{5}{7}$ α pertence ao 1.° ou 4.° Q

5.3 $\tan\alpha = -\frac{3}{5}$ α pertence ao 2.° ou 4.° Q

5.4 $\underbrace{\sin\alpha}_{>0}\cos\alpha < 0 \wedge \sin\alpha > 0$

Então $\cos\alpha < 0 \wedge \sin\alpha > 0$, logo

α pertence ao 2.° Q.

5.5 $0 \leqslant \sin\alpha \leqslant 1$ α pertence ao 1.° ou 2.° Q

5.6 $\cos\alpha\tan\alpha > 0 \Leftrightarrow (\cos\alpha > 0 \wedge \tan\alpha > 0)$

$\vee (\cos\alpha < 0 \wedge \tan\alpha < 0)$

$\Leftrightarrow (\alpha \in$ 1.° ou 4.° Q $\wedge\ \alpha \in$ 1.° ou 3.° Q$)$

$\vee (\alpha \in$ 2.° ou 3.° Q $\wedge\ \alpha \in$ 2.° ou 4.° Q$)$

$\Leftrightarrow \alpha \in$ 1.° Q $\vee\ \alpha \in$ 2.° Q

α pertence ao 1.° ou 2.° Q

5.7 $\frac{\overbrace{\cos^2\alpha}^{>0}}{\tan\alpha} > 0 \Leftrightarrow \tan\alpha > 0$

$\Leftrightarrow \alpha$ pertence ao 1.° ou 3.° Q

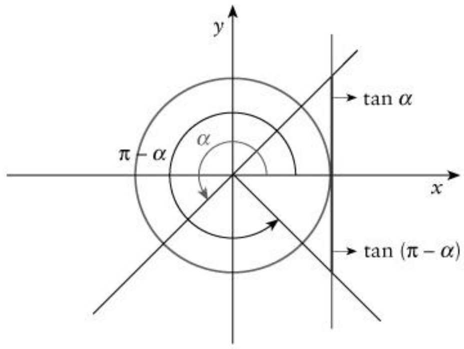
6.1 $f(\alpha) = (1 + \sin\alpha)^2 + \cos\alpha$ **Pág. 198**

$f(\alpha) = ?$

$\alpha \in$ 3.° Q $\tan\overbrace{(\pi - \alpha)}^{\in\ 4.°\ \text{Q}} = -3$

$-\tan\alpha = -3$

$\tan\alpha = 3$



$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} \Leftrightarrow 3 = \frac{\sin\alpha}{\cos\alpha}$

$\Leftrightarrow \sin\alpha = 3\cos\alpha$

Fórmula Fundamental de Trigonometria:

$\frac{\sin^2\alpha}{\cos^2\alpha} + \frac{\cos^2\alpha}{\cos^2\alpha} = \frac{1}{\cos^2\alpha}$

$\Leftrightarrow \tan^2\alpha + 1 = \frac{1}{\cos^2\alpha} \Leftrightarrow 3^2 + 1 = \frac{1}{\cos^2\alpha}$

$\Leftrightarrow \cos^2\alpha = \frac{1}{10} \Leftrightarrow \cos\alpha = \pm\sqrt{\frac{1}{10}}$

$\Leftrightarrow \cos\alpha = \pm\frac{1}{\sqrt{10}} \Leftrightarrow \cos\alpha = \pm\frac{\sqrt{10}}{10}$,

como $\alpha \in$ 3.° Q, logo, o co-seno é negativo,

portanto, $\cos\alpha = -\frac{\sqrt{10}}{10}$ como $\sin\alpha = 3\cos\alpha$

$\Leftrightarrow \sin\alpha = 3 \times \left(-\frac{\sqrt{10}}{10}\right)$

$\Leftrightarrow \sin\alpha = -\frac{3\sqrt{10}}{10}$

$f(\alpha) = (1+\sin\alpha)^2 + \cos\alpha$

$\Leftrightarrow f(\alpha) = \left(1 - \frac{3\sqrt{10}}{10}\right)^2 + \left(-\frac{\sqrt{10}}{10}\right)$

$= 1 - \frac{6\sqrt{10}}{10} + \frac{9\times 10}{100} - \frac{\sqrt{10}}{10}$

$= \underset{(10)}{\frac{1}{1}} - \frac{6\sqrt{10}}{10} + \frac{90}{100} - \frac{\sqrt{10}}{10}$

$= \frac{10}{10} - \frac{6\sqrt{10}}{10} + \frac{9}{10} - \frac{\sqrt{10}}{10} = \frac{19 - 7\sqrt{10}}{10}$

6.2 $\alpha \in 2.^\circ$ Q $\quad \sin\left(\frac{\pi}{2}+\alpha\right) = -\frac{1}{5}$

$\sin(\pi+\alpha) - \cos^2(\pi-\alpha) + \sin\left(\frac{3\pi}{2}-\alpha\right) +$

$+ \cos(\alpha - 7\pi)$

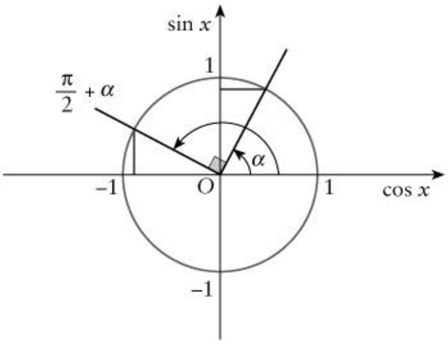
$= -\sin\alpha - (-\cos\alpha)^2 - \cos\alpha - \cos\alpha =$

$= -\sin\alpha - \cos^2\alpha - 2\cos\alpha$

$= -\frac{2\sqrt{6}}{5} - \left(-\frac{1}{5}\right)^2 - 2\times\left(-\frac{1}{5}\right)$

$= -\underset{(5)}{\frac{2\sqrt{6}}{5}} - \frac{1}{25} + \underset{(5)}{\frac{2}{5}}$

$= -\frac{10\sqrt{6}}{25} - \frac{1}{25} + \frac{10}{25} = \frac{-10\sqrt{6}+9}{25}$



Cálculo auxiliar

$\sin\left(\frac{\pi}{2}+\alpha\right) = \frac{1}{5} \Leftrightarrow \cos\alpha = \frac{1}{5}$

$\sin^2\alpha + \cos^2\alpha = 1 \longrightarrow$ Fórm. Fund. Trigonometria

$\sin^2\alpha + \left(\frac{1}{5}\right)^2 = 1 \Leftrightarrow \sin^2\alpha = 1 - \frac{1}{25}$

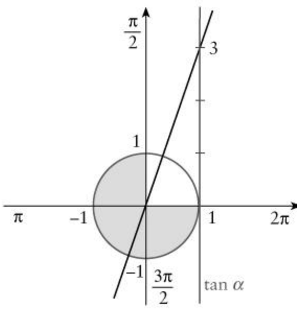
$\Leftrightarrow \sin^2\alpha = \frac{25}{25} - \frac{1}{25} \Leftrightarrow \sin^2\alpha = \frac{24}{25}$

$\Leftrightarrow \sin\alpha = \pm\sqrt{\frac{24}{25}} \Leftrightarrow \sin\alpha = \pm\sqrt{\frac{24}{5}}$

$\Leftrightarrow \sin\alpha = \pm\frac{2\sqrt{6}}{5}$, como $\alpha \in 2.^\circ$ Q,

então, $\sin\alpha = \frac{2\sqrt{6}}{5}$.

7.1 **Pág. 199**



$\tan\alpha = 3$ e $\alpha \in \left]\frac{\pi}{2}, 2\pi\right[$

$A(\alpha) = 1 + \cos\alpha - 2\sin\alpha = ?$

$\tan\alpha = 3$

como $\tan\alpha = 3 > 0$, logo $\alpha \in \left]\pi, \frac{3\pi}{2}\right[$

$\tan\alpha = 3 \quad \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = 3$

$\sin\alpha = 3\cos\alpha$

Fórmula fundamental da trigonometria:

$\sin^2\alpha + \cos^2\alpha = 1$

$(3\cos\alpha)^2 + \cos^2\alpha = 1 \Leftrightarrow 9\cos^2\alpha + \cos^2\alpha = 1$

$\Leftrightarrow 10\cos^2\alpha = 1 \Leftrightarrow \cos^2\alpha = \frac{1}{10}$

$\Leftrightarrow \cos\alpha = \pm\sqrt{\frac{1}{10}} \Leftrightarrow \cos\alpha = \pm\frac{1}{\sqrt{10}}$

$\Leftrightarrow \cos\alpha = \pm\frac{\sqrt{10}}{10}$, como $\alpha \in 3.^\circ$ Q,

logo, $\cos\alpha = -\frac{\sqrt{10}}{10} \qquad \sin\alpha = ?$

$\sin\alpha = 3\cos\alpha \Leftrightarrow \sin\alpha = 3\times\left(-\frac{\sqrt{10}}{10}\right)$

$\Leftrightarrow \sin\alpha = -\frac{3\sqrt{10}}{10}$

$A(\alpha) = 1 + \cos\alpha - 2\sin\alpha$

$\Leftrightarrow A(\alpha) = 1 - \frac{\sqrt{10}}{10} - 2\times\left(\frac{-3\sqrt{10}}{10}\right)$

$\Leftrightarrow A(\alpha) = 1 - \frac{\sqrt{10}}{10} + \frac{6\sqrt{10}}{10}$

$\Leftrightarrow A(\alpha) = 1 + \frac{5\sqrt{10}}{10} \Leftrightarrow A(\alpha) = 1 + \frac{\sqrt{10}}{2}$

7.2 $\cos(\pi+\theta) = -\frac{1}{4}, \ \theta \in]-\pi; 2\pi[$

$A(\theta) = \cos\left(\frac{\pi}{2}-\theta\right) - 3\tan(\theta - 7\pi)$

$= -\sin\theta - 3\tan(\theta)$

Sabe–se que: $\cos(\pi+\theta) = -\frac{1}{4}$

$\Leftrightarrow -\cos\theta = -\frac{1}{4}$

$\Leftrightarrow \cos\theta = \frac{1}{4}$, logo $\theta \in 4.^\circ$ Q

Cálculo de $\sin\theta$, aplicando a Fórmula Fundamental da Trigonometria:

$\sin^2(\theta) + \cos^2(\theta) = 1$

$\Leftrightarrow \sin^2(\theta) = 1 - \left(\frac{1}{4}\right)^2 \Leftrightarrow \sin^2\theta = 1 - \frac{1}{16}$

$\Leftrightarrow \sin^2\theta = \frac{15}{16} \Leftrightarrow \sin\theta = \pm\sqrt{\frac{15}{16}}$

$\Leftrightarrow \sin\theta = \pm\frac{\sqrt{15}}{4} \Leftrightarrow \sin\theta = -\frac{\sqrt{15}}{4}$,

porque $\theta \in 4.^\circ$ Q.

Cálculo de $\tan(\theta)$:

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{\sqrt{15}}{4}}{\frac{1}{4}} = -\sqrt{15}$

Assim, vem: $\sin\theta - 3\tan\theta$

$= \left(-\frac{\sqrt{15}}{4}\right) - 3(-\sqrt{15}) = \frac{-\sqrt{15}}{4} + 3\sqrt{15}$

$= \frac{-\sqrt{15}}{4} + \frac{12\sqrt{15}}{4} = \frac{11\sqrt{15}}{4}$

1. $-\frac{16\pi}{3} = \underbrace{-\frac{12}{3}\pi}_{\text{2 voltas completas no sentido negativo.}} - \frac{4}{3} \in 2.^\circ$ Q **Pág. 200**

R.: (B).

2. (A) $\tan 330° \neq \frac{\sqrt{3}}{3} \longrightarrow$ Pois a tangente no 4.° Q é negativa.

(B) $\sin(-60°) \neq \frac{\sqrt{3}}{2} \longrightarrow$ Pois o seno no 4.° Q é negativo.

(C) $\cos\frac{5\pi}{2} = \cos\frac{\pi}{2}$?

$$\cos\frac{5\pi}{2} = \cos\left(\frac{4\pi}{2}+\frac{\pi}{2}\right) = \cos\left(2\pi+\frac{\pi}{2}\right) = \cos\frac{\pi}{2}$$

Logo, a afirmação (C) é verdadeira.

R.: (C) .

3. $\theta \in 2.° \text{ Q}$

(A) $\underbrace{\underbrace{\sin\theta}_{>0}\;\underbrace{\cos\theta}_{<0}}_{<0} < 0$

(B) $\underbrace{\underbrace{\tan\theta}_{<0}\;\underbrace{\cos\theta}_{<0}}_{>0} > 0$

(C) $\underbrace{\underbrace{\sin^3\theta}_{>0}\;\underbrace{\cos\theta}_{<0}}_{<0} < 0$

(D) $\underbrace{\sin\theta}_{>0} > \underbrace{\cos\theta}_{<0}$

R.: (D) .

4. $\sin\left(\frac{3\pi}{2}-x\right) = -\frac{1}{5}$, $x \in 4.° \text{ Q}$

(A) $\cos x \neq -\frac{1}{5} \longrightarrow$ Porque $x \in 4.° \text{ Q}$, logo o co-seno é positivo.

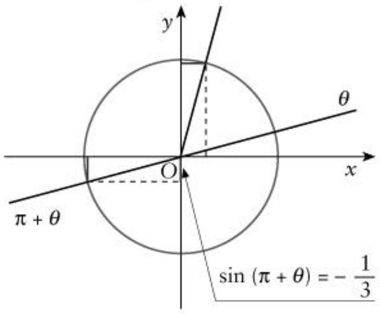
(B) $\sin x \neq \frac{1}{5} \longrightarrow$ Porque $x \in 4.° \text{ Q}$, logo o seno é negativo.

(C) $\cos x \neq \frac{1}{5} \longrightarrow$ Porque $\sin\left(\frac{3\pi}{2}-x\right) = -\frac{1}{5}$
$\Leftrightarrow -\cos x = -\frac{1}{5} \Leftrightarrow \cos x = \frac{1}{5}$.

(D) $\sin x \neq -\frac{1}{5} \longrightarrow$ Porque pela alínea anterior $\cos x = \frac{1}{5}$, e aplicando a Fórmula Fundamental da Trigonometria $\sin^2 x + \cos^2 x = 1$, o $\sin x \neq -\frac{1}{5}$.

R.: (C) .

5. $\sin(\pi+\theta) = -\frac{1}{3}$ e $\theta \in 2.° \text{ Q}$.



$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$

$\sin^2(\pi+\theta) + \cos^2(\pi+\theta) = 1$

$\left(-\frac{1}{3}\right)^2 + \cos^2(\pi+\theta) = 1$

$\Leftrightarrow \cos^2(\pi+\theta) = 1 - \frac{1}{9} \Leftrightarrow \cos^2(\pi+\theta) = \frac{8}{9}$

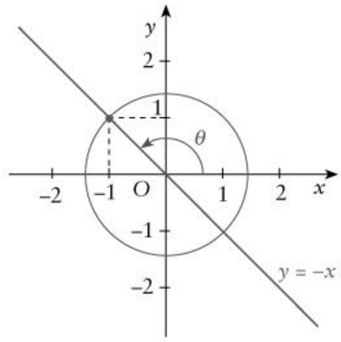
$\Leftrightarrow \cos(\pi+\theta) = \pm\sqrt{\frac{8}{9}}$

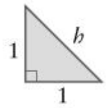
$\Leftrightarrow \cos(\pi+\theta) = \cancel{\pm}\frac{2\sqrt{2}}{3}$

Logo, $\cos\theta = -2\frac{\sqrt{2}}{3} = \sin\left(\frac{\pi}{2}-\theta\right)$

R.: (C) .

6. $y = -x$





Cálculo auxiliar
Teorema de Pitágoras:
$h^2 = 1^2 + 1^2$
$\Leftrightarrow h = \cancel{\pm}\sqrt{2}$

$\sin\theta = \frac{1}{\sqrt{2}} \Leftrightarrow \sin\theta = \frac{\sqrt{2}}{2}$

$\sin(-\theta) = -\frac{\sqrt{2}}{2}$, logo (A) fica excluída

$\cos\theta = \frac{-1}{\sqrt{2}} \Leftrightarrow \cos\theta = -\frac{\sqrt{2}}{2}$

$\cos(-\theta) = -\frac{\sqrt{2}}{2} \longrightarrow$ confirma-se a hipótese (B)

R.: (B) .

7. $\tan\theta = \frac{a}{b}$, como $a > 0$ e $b > 0$

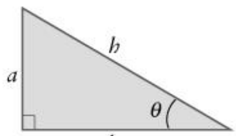
$\frac{a}{b} > 0$, ou seja $\tan\theta > 0$,

logo, $\theta \in 1.°$ ou $3.° \text{ Q}$.

Se $\theta \in 3.° \text{ Q}$, tem-se $\sin\theta < 0$ e $\cos\theta < 0$, o que exclui (A) e (B) .

(C) $\sin\theta = \pm\frac{a}{\sqrt{a^2+b^2}}$

$\tan\theta = \frac{a}{b}$



Cálculo auxiliar
Teorema de Pitágoras:
$h^2 = a^2 + b^2$
$\Leftrightarrow h^2 = \sqrt{a^2+b^2}$

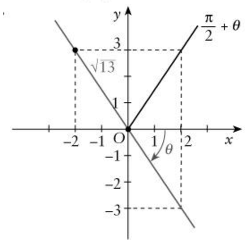
$\sin\theta = \frac{\text{cat. op. } \theta}{\text{hip.}} \Leftrightarrow \sin\theta = \frac{a}{\sqrt{a^2+b^2}}$

Como $\theta \in 1.° \text{ Q}$ ou $\theta \in 2.° \text{ Q}$:

$\sin\theta = \pm\frac{a}{\sqrt{a^2+b^2}}$

R.: (C)

8.



$\sin\theta = \frac{-3}{\sqrt{13}} = \frac{-3\sqrt{13}}{13}$

$\cos\theta = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$

$\tan\theta = \frac{-3}{2}$

(A) $\sin\left(\frac{\pi}{2}+\theta\right)=\frac{\sqrt{13}}{13}$?

$\sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta=\frac{2\sqrt{13}}{13}$

$\therefore \sin\left(\frac{\pi}{2}+\theta\right)=\frac{\sqrt{13}}{13} \longrightarrow$ falso

(B) $\cos(\pi-\theta)=\frac{2}{3}$

$\cos(\pi-\theta)=-\cos\theta=-\frac{2\sqrt{13}}{13} \longrightarrow$ falso

(C) $\tan(\theta-\pi)=\frac{3}{2}$

$\tan(\theta-\pi)=\tan\theta=-\frac{3}{2} \longrightarrow$ falso

(D) $\cos(\pi+\theta)=-\frac{2\sqrt{13}}{13}$

$\cos(\pi+\theta)=-\cos\theta=-\frac{2\sqrt{13}}{13} \longrightarrow$ verdadeiro

R.: **(D)** .

Pág. 201

9.1 $1015° = 2\times 360° + 295°$

$1015° \in 4.°$ Q

9.2 $2215° = 6\times 360° + 55°$

$2215° \in 1.°$ Q

9.3 $-48 \in 4.°$ Q

9.4 $-6015° = 16\times(-360°) - 255°$

$-6015° \in 2.°$ Q

10.1 $\cos\frac{\pi}{2} - \tan\pi + \sin\pi = 0 - 0 + 0 = 0$

10.2 $\cos(-675°)+\tan(225°)-\sin(-270°)$

$=\cos(-360°-315°)+\tan(180°+45°)-\sin 90°$

$=\cos(-315°)+\tan 45° - 1$

$=\cos 45° + 1 - 1 = \frac{\sqrt{2}}{2}+0=\frac{\sqrt{2}}{2}$

10.3 $\sin\left(\frac{19\pi}{3}\right)\times\cos\left(-\frac{25\pi}{6}\right)$

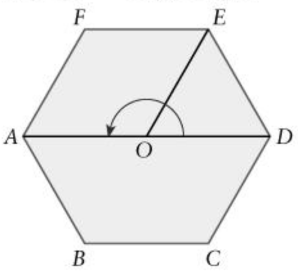
$=\sin\left(\frac{18\pi}{3}+\frac{\pi}{3}\right)\times\cos\left(-\frac{24\pi}{6}-\frac{\pi}{6}\right)$

$=\sin\left(6\pi+\frac{\pi}{3}\right)\times\cos\left(-4\pi-\frac{\pi}{6}\right)$

$=\sin\left(\frac{\pi}{3}\right)\times\cos\left(-\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}\times\cos\frac{\pi}{6}$

$=\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}=\frac{(\sqrt{3})^2}{4}=\frac{3}{4}$

11. $[ABCDEF] \longrightarrow$ hexágono regular



Expressão geral da amplitude dos ângulos

11.1. $\dot{O}D$ como lado origem e $\dot{O}A$ como l

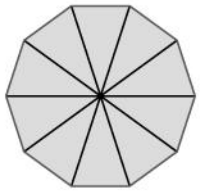
midade

$x=\pi+2k\pi\,,\ k\in\mathbb{Z}$

11.2. $\dot{O}D$ como lado origem e $\dot{O}E$ lado extr

$x=\frac{\pi}{3}+2k\pi\,,\ k\in\mathbb{Z}$

12. Ângulo interno de 1 decágono regular.



$\frac{360°}{10}=36°$

$180°-36°=144°$

$\frac{144}{2}=72°$

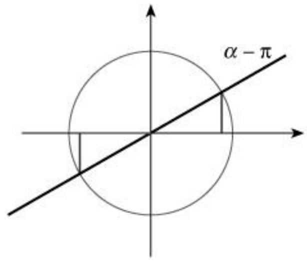
$72°\times 2 = 144°$

R.: $144° \in 2.°$ Q

13. $\sin(\alpha-\pi)=\frac{1}{3}\,,\ \alpha\in 3.°$ Q

$\tan(\pi-\alpha)+2\cos(5\pi-\alpha)=?$

$\sin(\alpha-\pi)=\frac{1}{3} \Leftrightarrow \sin\alpha=-\frac{1}{3}$



$\cos^2\alpha+\sin^2\alpha=1 \Leftrightarrow \cos^2\alpha+\left(-\frac{1}{3}\right)^2=1$

$\Leftrightarrow \cos^2\alpha=\underset{(9)}{\frac{1}{1}}-\frac{1}{9} \Leftrightarrow \cos^2\alpha=\frac{8}{9}$

$\Leftrightarrow \cos\alpha\pm\sqrt{\frac{8}{9}} \Leftrightarrow \cos\alpha=\pm\frac{\sqrt{8}}{3}$

$\Leftrightarrow \cos\alpha=\pm\frac{2\sqrt{2}}{3}$. Como $\alpha\in 3.°$ Q

$\cos\alpha=-\frac{2\sqrt{2}}{3}$

$\tan(\pi-\alpha)+2\cos(5\pi-\alpha)$

$=-\tan\alpha+2\cos(\pi-\alpha)=-\frac{\sin\alpha}{\cos\alpha}-2\cos\alpha$

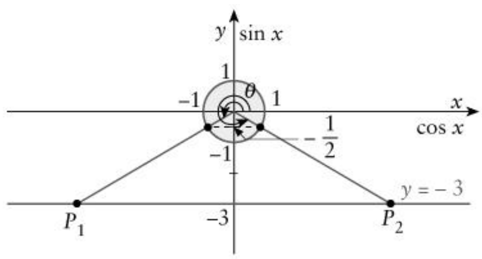
$=-\frac{-\frac{1}{3}}{-\frac{2\sqrt{2}}{3}}-2\times\left(-\frac{2\sqrt{2}}{3}\right)$

$=-\frac{3}{3\times 2\sqrt{2}}+\frac{4\sqrt{2}}{3}=-\frac{1}{2\sqrt{2}}+\frac{4\sqrt{2}}{3}$

$=-\frac{\sqrt{2}}{4}+\frac{4\sqrt{2}}{3}=\frac{13\sqrt{2}}{12}$

14. $P \curvearrowright (x\,,\,-3)$

$P\in$ lado extremidade do ângulo θ e $\sin\theta=-\frac{1}{3}$

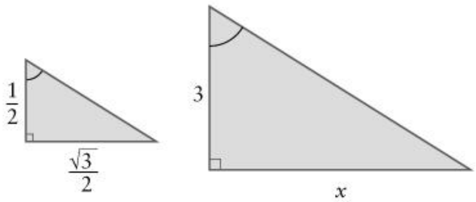


$\sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6} + 2k\pi \;\vee\; \theta = -\frac{\pi}{6} + 2k\pi\,,\; k \in \mathbb{Z}$

$\cos\theta = ?\qquad \cos\theta = \frac{\sqrt{3}}{2} \;\vee\; \cos\theta = -\frac{\sqrt{3}}{2}$

Os seguintes triângulos são semelhantes:



Logo: $\frac{3}{\frac{1}{2}} = \frac{x}{\frac{\sqrt{3}}{2}} \Leftrightarrow 6 = \frac{2x}{\sqrt{3}} \Leftrightarrow \frac{6\sqrt{3}}{2} = x$

$\Leftrightarrow\; x = 3\sqrt{3}$

Existem dois pontos que satisfazem as condições do problema, são:

$P_1 \curvearrowright (3\sqrt{3}\,;\,-3)$ ou $P_2 \curvearrowright (-3\sqrt{3}\,;\,-3)$

R.: $x = 3\sqrt{3} \;\vee\; x = -3\sqrt{3}$

15. 7. $Q \curvearrowright (-3\,,\,y)$ pertence ao lado extremidade de θ:

$\cos\theta = -\frac{4}{13}$

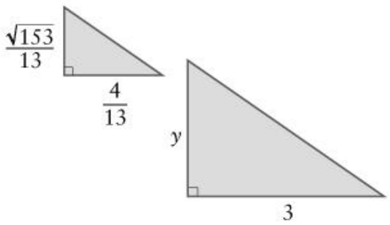
Fórmula Fundamental da Trigonometria:

$\sin^2\theta + \cos^2\theta = 1 \Leftrightarrow \sin^2\theta + \left(-\frac{4}{13}\right)^2 = 1$

$\Leftrightarrow \sin^2\theta = 1 - \frac{16}{169} \Leftrightarrow \sin^2\theta = \frac{153}{169}$

$\Leftrightarrow \sin\theta = \pm\sqrt{\frac{153}{169}} \Leftrightarrow \sin\theta = \pm\frac{\sqrt{153}}{13}$

Os seguintes triângulos são semelhantes:



$\frac{3}{\frac{4}{13}} = \frac{y}{\frac{\sqrt{153}}{13}} \Leftrightarrow \frac{3}{4} = \frac{y}{\sqrt{153}}$

$\Leftrightarrow \frac{3\times\sqrt{153}}{4} = y \Leftrightarrow y = \frac{3\sqrt{153}}{4}$

$y = \frac{3\sqrt{153}}{4} \;\vee\; y = -\frac{3\sqrt{153}}{4}$

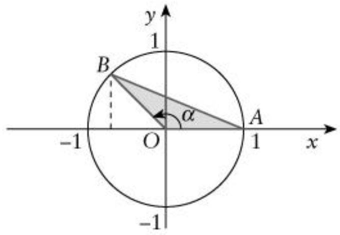
16. $\frac{\sin^2 x}{\cos x \tan x} < 0$

Como $\sin^2 x \geqslant 0$, logo, para que: $\frac{\sin^2 x}{\cos x \tan x} < 0$, então $\cos x \tan x$ tem de ser menor que zero, ou seja, $\cos x \tan x < 0$.

Para que $\cos x \tan x < 0$, o co-seno e a tangente têm de ter sinais contrários. Ora, isto acontece no 3.° quadrante onde o co-seno é negativo e a tangente é positiva. O mesmo sucede no 4.° quadrante, embora aí o co-seno seja positivo e a tangente negativa.

Então, pode-se concluir que isto só se verifica se x pertence ao 3.° ou 4.° quadrantes.

17. $\alpha = A\hat{O}B$



$A_\Delta = \frac{\text{base} \times \text{altura}}{2}$

$A_{\Delta[AOB]} = \frac{\overline{OA} \times \sin\alpha}{2} \Leftrightarrow A_{\Delta[AOB]} = \frac{1\times\sin\alpha}{2}$

$\Leftrightarrow A_{\Delta[AOB]} = \frac{\sin\alpha}{2}$