Capítulo 6

1. Introdução ao estudo da trigonometria. Razões trigonométricas no triângulo rectângulo (revisão)

1. **Pág. 175**

$180° — \pi$ rad $\quad x \times \pi = 180° \times 1 \Leftrightarrow$

$x — 1$ rad $\quad \Leftrightarrow x = \dfrac{180°}{\pi} \Leftrightarrow x \approx 57{,}296°$

```
180/π
        57.29577951
(Ans-57)*60
        17.74677078
(Ans-17)*60
         44.8062471
```

ou

```
180/π
        57.29577951
Ans▸DMS
        57°17'44.806"
```

Um radiano corresponde aproximadamente a 57 graus, 17 minutos e 45 segundos.

2.1 a) **Pág. 176**

$180° — \pi$ rad

$62° — x$ rad

$x \times 180° = \pi \times 62° \Leftrightarrow x = \dfrac{\pi \times 62°}{180°}$

$\Leftrightarrow x \approx 1{,}08$ rad

$62°$ correspondem a $1{,}08$ rad, aproximadamente.

b) $180° — \pi$ rad

$300° — x$ rad

$x \times 180° = \pi \times 300° \Leftrightarrow x = \dfrac{\pi \times 300°}{180°}$

$\Leftrightarrow x \approx 5{,}24$ rad

$300°$ correspondem a $5{,}24$ rad, aproximadamente.

c) $1° — 60'$

$x° — 30'$

$x \times 60 = 1 \times 30 \Leftrightarrow x = \dfrac{30}{60} \Leftrightarrow x = 0{,}5°$

Então, $15°\ 30' = 15{,}5°$

$180° — \pi$ rad

$15{,}5° — x$ rad

$x \times 180° = \pi \times 15{,}5° \Leftrightarrow x = \dfrac{\pi \times 15{,}5°}{180°}$

$\Leftrightarrow x \approx 0{,}27$ rad

$15°\ 30'$ correspondem a $0{,}27$ rad, aproximadamente.

d) $1' — 60''$

$x — 13''$

$x = \dfrac{13}{60} \Leftrightarrow x = 0{,}22'$ (2 c. d.)

$12°\ 10'\ 13'' \approx 12°\ 10{,}22'$

$1° — 60'$

$x° — 10{,}22'$

$x = \dfrac{10{,}22}{60} \Leftrightarrow x \approx 0{,}17°$ (2 c. d.)

$12°\ 10'\ 13'' \approx 12{,}17°$

$180° — \pi$ rad

$12{,}17° — x$ rad

$x = \dfrac{\pi \times 12{,}17°}{180°} \Leftrightarrow x = 0{,}21$ rad (2 c. d.)

$12°\ 10'\ 13''$ correspondem a $0{,}21$ rad, aproximadamente.

$12°\ 10'\ 13''$ correspondem a $0{,}21$ rad, aproximadamente.

2.2 a) $180° — \pi$ rad

$x — \dfrac{5\pi}{3}$ rad

$x = \dfrac{180 \times \frac{5\pi}{3}}{\pi} \Leftrightarrow x = \dfrac{180 \times 5\cancel{\pi}}{3\cancel{\pi}}$

$\Leftrightarrow x = 60 \times 5 \Leftrightarrow x = 300°$;

b) $180° — \pi$ rad

$x — \dfrac{5\pi}{3}$ rad

$x = \dfrac{180 \times \frac{5\pi}{3}}{\pi} \Leftrightarrow x = \dfrac{180 \times 5\cancel{\pi}}{3\cancel{\pi}}$

$\Leftrightarrow x = 60 \times 5 \Leftrightarrow x = 300°$;

c) Sabemos que $3{,}14$ rad são aproximadamente $180°$, no entanto, calcula-se:

$180° — \pi$ rad

$x° — 3{,}14$ rad

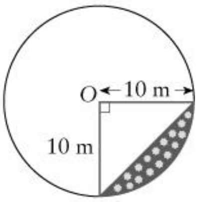
$x = \dfrac{180° \times 3{,}14}{\pi} \Leftrightarrow x = 179{,}91$ (2 c. d.);

d) $180° — \pi$ rad

$x° — 0{,}2$ rad

$x = \dfrac{180° \times 0{,}2}{\pi} \Leftrightarrow x = 11{,}46°$ (2 c. d.).

3. **Pág. 177**

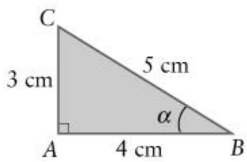


$\boxed{A_{\odot} = \pi\, r^2} \Leftrightarrow A_{\odot} = 100\pi\ \text{m}^2$

$A_{\triangledown} = \dfrac{A_{\odot}}{4} = \dfrac{100\pi}{4} = 25\pi\ \text{m}^2$

$\boxed{A_{\triangledown} = \dfrac{b \times r}{2}} \Leftrightarrow A_{\triangledown} = \dfrac{10 \times 10}{2} = 50\ \text{m}^2$

4.1 **Pág. 178**



Razões trigon. de $\alpha = ?$

Teorema de Pitágoras

$\overline{BC}^2 = \overline{AC}^2 + \overline{AB}^2$

$\overline{BC}^2 = 3^2 + 4^2$

$\overline{BC}^2 = 25$

$\overline{BC} = \pm\sqrt{25}$

$\overline{BC} = \cancel{\pm}\, 5$

$\sin \alpha = \dfrac{\text{cat. op.}}{\text{hip.}} = \dfrac{3}{5} \qquad \cos \alpha = \dfrac{\text{cat. adj.}}{\text{hip.}} = \dfrac{4}{5}$

$\tan \alpha = \dfrac{\text{cat. op.}}{\text{cat. adj.}} = \dfrac{3}{4}$

4.2 Como $\sin\alpha$ é igual ao quociente entre a medida do cateto oposto a α e a medida da hipotenusa, então é imediato que: $0 < \sin\alpha < 1$, pois a hipotenusa é o lado de maior comprimento de um triângulo rectângulo. Para $0 < \cos\alpha < 1$, a razão é análoga, em vez do cateto oposto, temos o adjacente.

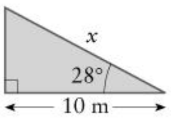
5 **Pág. 179**

$$\frac{\sin\frac{\pi}{6}\times\cos\frac{\pi}{3}}{\tan\frac{\pi}{4}}\times\sin^2\frac{\pi}{6}$$

$$=\frac{\frac{1}{2}\times\frac{1}{2}}{1}\times\left(\frac{1}{2}\right)^2=\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$$

Note que: $\sin^2\frac{\pi}{6}=\left(\sin\frac{\pi}{6}\right)^2$.

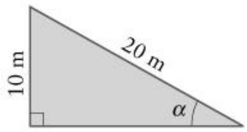
6.1 **Pág. 180**



$$\cos\alpha=\frac{\text{cat. adj. }\alpha}{\text{hip.}}\Leftrightarrow\cos 28°=\frac{10}{x}$$

$$\Leftrightarrow x=\frac{10}{\cos 28°}\Leftrightarrow x=11{,}33\text{ m (2 c. d.)}$$

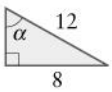
6.2



$$\sin\alpha=\frac{\text{cat. op. a }\alpha}{\text{hip.}}\Leftrightarrow\sin\alpha=\frac{10}{20}$$

$$\Leftrightarrow\sin\alpha=\frac{1}{2}\Leftrightarrow\alpha=30°$$

6.3



Sabemos que $\sin\alpha=\frac{\text{cat. op.}}{\text{hip.}}$

$$\Leftrightarrow\sin\alpha=\frac{8}{12}\Leftrightarrow\sin\alpha=\frac{2}{3}$$

$$\sin^2\alpha+\cos^2\alpha=1$$

$$\Leftrightarrow\cos^2\alpha=1-\frac{4}{9}\Leftrightarrow\cos^2\alpha=\frac{5}{9}$$

$$\Leftrightarrow\cos\alpha=\frac{\sqrt{5}}{3}$$

$$\tan\alpha=\frac{\frac{2}{3}}{\frac{\sqrt{5}}{3}}=\frac{2\sqrt{5}}{5}$$

R.: $\tan\alpha=\frac{2\sqrt{5}}{5}$

7.1 **Pág. 181**

$\cos\alpha=\frac{1}{5}=0{,}2\longrightarrow$ valor exacto

$$\cos\alpha=\frac{1}{5}$$

$$\sin^2\alpha+\cos^2\alpha=1$$

$$\sin^2\alpha+\left(\frac{1}{5}\right)^2=1\Leftrightarrow\sin^2\alpha+\frac{1}{25}=1$$

$$\Leftrightarrow\sin^2\alpha=\frac{25}{25}-\frac{1}{25}\Leftrightarrow\sin^2\alpha=\frac{24}{25}$$

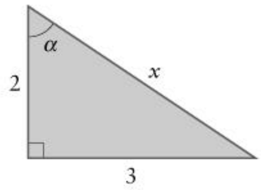
$$\Leftrightarrow\sin\alpha=\pm\sqrt{\frac{24}{25}}\Leftrightarrow\sin\alpha=\pm\frac{\sqrt{24}}{5}$$

$\Leftrightarrow\sin\alpha=\cancel{\pm}\frac{2\sqrt{6}}{5}\longrightarrow$ valor exacto

porque α é agudo

$\sin\alpha\approx 0{,}98$ (2 c. d.)

7.2 $\tan\alpha=\frac{3}{2}$



Valor exacto de $\sin\alpha$:

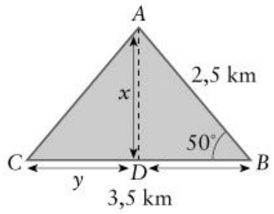
$$x^2=2^2+3^2\Leftrightarrow x^2=4+9\Leftarrow x=\sqrt{13}$$

$$\sin\alpha=\frac{\text{cat. op.}}{\text{hip.}}=\frac{3}{\sqrt{13}}=\frac{3\sqrt{13}}{13}$$

O valor exacto de $\sin\alpha$ é $\frac{3\sqrt{13}}{13}$.

O valor aproximado de $\sin\alpha$ é 0,83 (2 c. d.).

8 **Pág. 182**



Sendo $\overline{CD}=y$ e $\overline{AD}=x$, pretendemos determinar $\overline{AC}=\sqrt{x^2+y^2}$.

Observando a figura, tem-se:

$$\cos 50°=\frac{\overline{DB}}{2{,}5}\Leftrightarrow\overline{DB}=2{,}5\cos(50°)$$

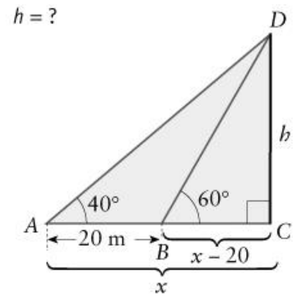
$$y=3{,}5-2{,}5\cos(50°)$$

$$\sin 50°=\frac{x}{2{,}5}\Leftrightarrow x=2{,}5\sin 50°$$

Então:

$$\overline{AC}^2=x^2+y^2$$

9.1 **Pág. 183**



$\tan \alpha = \dfrac{\text{cat. op. } \alpha}{\text{cat. adj. } \alpha}$

$\begin{cases} \tan 40° = \dfrac{h}{x} \\ \tan 60° = \dfrac{h}{x - 20} \end{cases} \Leftrightarrow \begin{cases} h = x \tan 40° \\ \sqrt{3}x - 20\sqrt{3} = h \end{cases}$

$\Leftrightarrow \begin{cases} \_\_\_\_\_\_ \\ \sqrt{3}x = h + 20\sqrt{3} \end{cases} \Leftrightarrow \begin{cases} \_\_\_\_\_\_ \\ x = \dfrac{h + 20\sqrt{3}}{\sqrt{3}} \end{cases}$

$\Leftrightarrow \begin{cases} h = \left(\dfrac{h}{\sqrt{3}} + 20\right) \tan 40° \\ x = \dfrac{h}{\sqrt{3}} + 20 \end{cases}$

$\Leftrightarrow \begin{cases} h = \dfrac{\tan 40° \times h}{\sqrt{3}} + 20 \times \tan 40° \\ \_\_\_\_\_\_ \end{cases}$

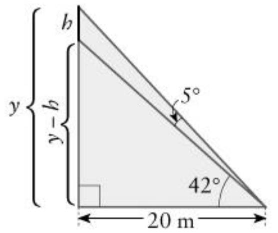
$\Rightarrow \sqrt{3}h = h \tan 40° + 20\sqrt{3} \tan 40°$

$\Leftrightarrow h\,(\sqrt{3} - \tan 40°) = 20\sqrt{3} \tan 40°$

$\Leftrightarrow h = \dfrac{20\sqrt{3} \tan 40°}{\sqrt{3} - \tan 40°}$

$\Leftrightarrow h \approx 32{,}55$ m

9.2



$\tan \alpha = \dfrac{\text{cat. op. } \alpha}{\text{cat. adj. } \alpha}$

$\begin{cases} \tan 42° = \dfrac{y - h}{20} \\ \tan 47° = \dfrac{y}{20} \end{cases} \Leftrightarrow \begin{cases} \_\_\_\_\_\_ \\ y = 20 \times \tan 47° \end{cases}$

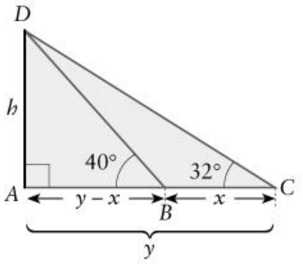
$\Leftrightarrow \begin{cases} \_\_\_\_\_\_ \\ y = 20 \tan 47° \end{cases} \Leftrightarrow \begin{cases} \tan 42° = \dfrac{20 \tan 47° - h}{20} \\ \_\_\_\_\_\_ \end{cases}$

$\Rightarrow 20 \tan 42° = 20 \tan 47° - h$

$\Leftrightarrow h = 20 \tan 47° - 20 \tan 42°$

$\Leftrightarrow h \approx 3{,}44$ m

9.3



$\tan \alpha = \dfrac{\text{cat. op. } \alpha}{\text{cat. adj. } \alpha}$

$\begin{cases} \tan 32° = \dfrac{h}{y} \\ \tan 40° = \dfrac{h}{y - x} \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} y = \dfrac{h}{\tan 32°} \\ \tan 40° = \dfrac{h}{\dfrac{h}{\tan 32°} - \underset{(\tan 32°)}{\dfrac{x}{1}}} \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} \_\_\_\_\_\_ \\ \tan 40° = \dfrac{h}{\dfrac{h - \tan 32° x}{\tan 32°}} \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} \_\_\_\_\_\_ \\ \tan 40° = \dfrac{\tan 32° \, h}{h - \tan 32° \, x} \end{cases}$

$\Leftrightarrow \begin{cases} \_\_\_\_\_\_ \\ (h - \tan 32° \, x) \times \tan 40° = \tan 32° \, h \end{cases}$

$\Leftrightarrow \begin{cases} \_\_\_\_\_\_ \\ \tan 40° \, h - \tan 40° \times \tan 32° \, x - \tan 32° \, h = \end{cases}$

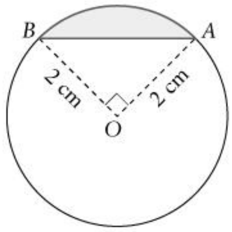
$\Leftrightarrow \begin{cases} \_\_\_\_\_\_ \\ (\tan 40° - \tan 32°) \, h = \tan 40° \times \tan 32° \end{cases}$

$\Leftrightarrow \begin{cases} \_\_\_\_\_\_ \\ h = \dfrac{\tan 40° \times \tan 32°}{\tan 40° - \tan 32} \, x \end{cases}$

Pág. 184

1. Raio: 2 cm

$[AB] \longrightarrow$ corda da circunferência ; $A\hat{O}P = \dfrac{\pi}{2}$



$A_{\text{parte colorida}} = ?$

$A_{\text{parte colorida}} = A_{\text{sector circular}} - A_{\triangle}$

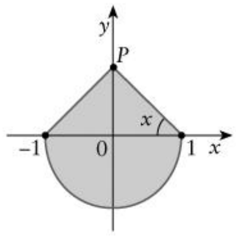
$A_{\text{sector circular}} = \dfrac{r^2 \alpha}{2} = \dfrac{2^2 \times \dfrac{\pi}{2}}{2} = \dfrac{2\pi}{2} = \pi \text{ cm}^2$

$A_{\triangle} = \dfrac{b \times h}{2} = \dfrac{2 \times 2}{2} = \dfrac{4}{2} = 2 \text{ cm}^2$

$A_{\text{parte colorida}} = (\pi - 2) \text{ cm}^2$

R.: **(D)**.

2.



$A_{\text{parte colorida}} = A_{\smile} + A_{\triangle}$

$A_{\smile} = \dfrac{A_{\odot}}{2} = \dfrac{\pi r^2}{2} = \dfrac{\pi}{2}$ (u. a.)

$A_{\triangle} = \dfrac{b \times h}{2} = \dfrac{2h}{2} = h$

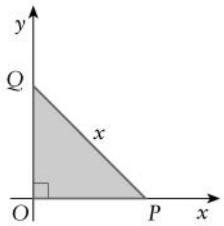
$h = ?$ $\tan x = \dfrac{h}{1} \Leftrightarrow h = \tan x$

$A_{\triangle} = \tan x$ (u. a.)

$A_{\text{parte colorida}} = \dfrac{\pi}{2} + \tan x$ (u. a.)

R.: **(C)**.

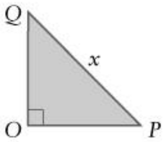
3.



$A_{\triangle [OPQ]} = \dfrac{b \times h}{2} = \dfrac{\overline{OP} \times \overline{OQ}}{2} = \dfrac{\overline{OP}^2}{2}$

$A_{\triangle [OPQ]} = \dfrac{\overline{OP}^2}{2}$

Pelo Teorema de Pitágoras:



$x^2 = \overline{OP}^2 + \overline{OP}^2$

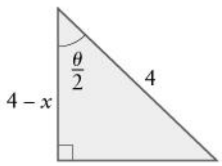
$\Leftrightarrow \; x^2 = 2\,\overline{OP}^2$

$\overline{OP}^2 = \dfrac{x^2}{2}$, logo:

$A_{\triangle [OPQ]} = \dfrac{\overline{OP}^2}{2} = \dfrac{\frac{x^2}{2}}{2} \Leftrightarrow A_{\triangle [OPQ]} = \dfrac{x^2}{4}$

R.: **(C)** .

4.



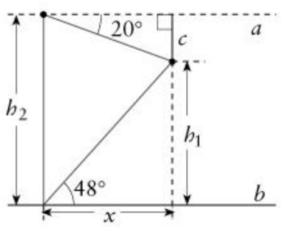
$\cos\left(\dfrac{\theta}{2}\right) = \dfrac{4-x}{4}$

$4\cos\left(\dfrac{\theta}{2}\right) = 4 - x$

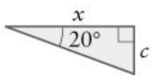
$x = 4 - 4\cos\left(\dfrac{\theta}{2}\right)$

R.: **(B)** .

5.



$a \,//\, b$



$h_2 - h_1 = ?$ (em função de x)

$h_2 - h_1 = c$

$\tan\alpha = \dfrac{\text{cat. op. } \alpha}{\text{cat. adj. } \alpha}$

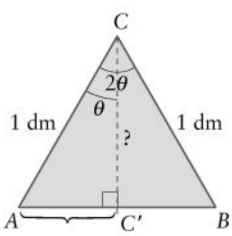
$\tan 20° = \dfrac{c}{x}$, $x \neq 0$

$c = x \tan 20°$, ou seja:

$h_2 - h_1 = x \tan 20°$ com $x \neq 0$

R.: **(A)** .

6.

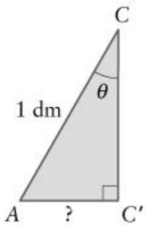


$A\hat{C}B = 2\theta$

$A_{\triangle [ABC]} = ?$

(em função de

$A_{\triangle [ABC]} = \dfrac{b \times h}{2}$



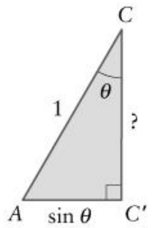
$\sin\theta = \dfrac{\text{cat. op.}}{\text{hip.}} \Leftrightarrow$

$\Leftrightarrow \sin\theta = \dfrac{x}{1} \Leftrightarrow$

$\Leftrightarrow x = \sin\theta$

$b = \overline{AB} = 2x = 2\sin\theta$

$b = 2\sin\theta$



Teorema de Pitágoras:

$1^2 = \sin^2\theta + h^2 \Leftrightarrow$

$\Leftrightarrow h^2 = 1^2 - \sin^2\theta$

$\Leftrightarrow h^2 = \cos^2\theta \Leftrightarrow$

$\Leftrightarrow h = \cos\theta$

$A_{\triangle [ABC]} = \dfrac{\cancel{2}\sin\theta \times \cos\theta}{\cancel{2}} = \sin\theta\cos\theta$

R.: **(A)** .

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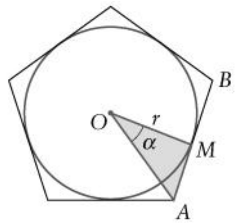
7. **Pág. 185**



$a \,//\, b$

$P_{\text{O}} = 5 \times \overline{AB}$

$\alpha = \dfrac{360°}{10} = 36°$

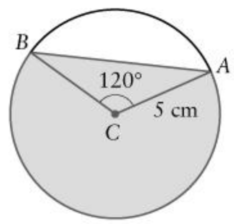
$\tan\alpha = \dfrac{\text{cat. op. } \alpha}{\text{cat. adj. } \alpha}$

$\tan 36° = \dfrac{\overline{AM}}{r} \Leftrightarrow \overline{AM} = r\tan 36°$

$\overline{AB} = 2\,\overline{AM} = 2 \times r\tan 36°$

Então, $P_{\text{O}} = 5 \times \overline{AB} = 5 \times 2 \times r\tan 36° = 10\,r\tan 36°$

8.



8.1 $S = r\,\alpha$, com α em radianos

Comprimento do arco raio ângulo

$\alpha = 120° = 180° - 60°$

$\alpha = \left(\pi - \dfrac{\pi}{3}\right)$ rad

$\alpha = \dfrac{2\pi}{3}$ rad

Comp. $AB = 5 \times \dfrac{2\pi}{3} = \dfrac{10\pi}{3}$ cm

8.2 $A_{\text{Sector } AB} = ?$

$A_{\text{Sector circ.}} = \dfrac{r^2\,\alpha}{2} \Leftrightarrow A = \dfrac{5^2 \times \frac{2\pi}{3}}{2}$

$\Leftrightarrow A = \dfrac{\frac{50\pi}{3}}{2} \Leftrightarrow A = \dfrac{25\pi}{3}\ \text{cm}^2$

8.3 $A_{\text{Parte colorida}} = A_{\text{Sect. circ. } BA} + A_{\triangle\,[ABC]}$

$A_{\text{Sect. circ. } BA} = \dfrac{r^2\,\alpha}{2}$

$\alpha = ?$

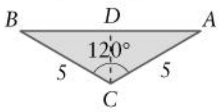
$\alpha = \underset{(3)}{\dfrac{2\pi}{1}} - \dfrac{2\pi}{3} = \dfrac{6\pi}{3} - \dfrac{2\pi}{3} = \dfrac{4\pi}{3}$

$A_{\text{Sector circ. } BA} = \dfrac{5^2 \times \frac{4\pi}{3}}{2}$

$\Leftrightarrow A_{\text{Sector circ. } BA} = \dfrac{\frac{100\pi}{3}}{2}$

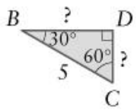
$\Leftrightarrow A_{\text{Sector circ. } BA} = \dfrac{100\pi}{6} = \dfrac{50\pi}{3}\ \text{cm}^2$

$A_{\triangle\,[ACB]} = ?$



$\overline{BD} = ?$

$\overline{DC} = ?$



$\overline{BD} = ?$

$\cos\alpha = \dfrac{\text{cat. adj. } \alpha}{\text{hip.}}$

$\Leftrightarrow \cos 30° = \dfrac{\overline{BD}}{5} \Leftrightarrow \overline{BD} = 5 \cos 30°$

$\Leftrightarrow \overline{BD} \approx \dfrac{5\sqrt{3}}{2}\ \text{cm}$

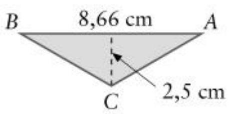
logo, $\overline{BA} = 5\sqrt{3}$ cm

$\overline{DC} = ?$

$\cos 60° = \dfrac{\overline{DC}}{5} \Leftrightarrow \overline{DC} = 5 \cos 60°$

$\Leftrightarrow \overline{DC} = 2{,}5$ cm

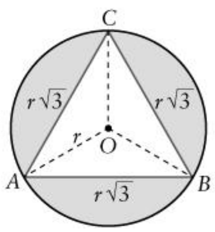
$A_{\triangle\,[ACB]} = \dfrac{b \times h}{2}$



$A_{\triangle\,[ACB]} = \dfrac{5\sqrt{3} \times 2{,}5}{2} = 6{,}25\sqrt{3}\ \text{cm}^2$

$A_{\text{parte colorida}} = \dfrac{50\pi}{3} + 6{,}25\sqrt{3}\ \text{cm}^2$

9. Triângulo equilátero $[ABC]$ inscrito no círculo de raio r e centro O.

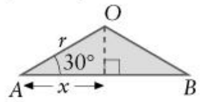


9.1 $P_{\triangle\,[ABC]}(r) = 3r\sqrt{3}$ (em função de r)

$P_{\triangle\,[ABC]} = 3 \times \overline{AB}$

$\overline{AB} = ?$

$\overline{AB} = 2x$



$\cos\alpha = \dfrac{\text{cat. adj. } \alpha}{\text{hip.}} \Leftrightarrow \cos 30° = \dfrac{x}{r}$

$\Leftrightarrow x = r\dfrac{\sqrt{3}}{2}$

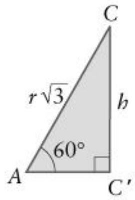
$\overline{AB} = 2x = 2 \times r\dfrac{\sqrt{3}}{2} = r\sqrt{3}$

$P_{\triangle\,[ABC]}(r) = 3 \times \overline{AB} = 3 \times r\sqrt{3} = 3\,r\sqrt{3}$ c. q. d.

9.2 $\dfrac{\text{Médida área da parte colorida}}{\text{Medida da área do triângulo } [ABC]} = \dfrac{4\pi\sqrt{3} - 9}{9}$

$A_{\triangle\,[ABC]} = \dfrac{b \times h}{2}$ $\quad b = \text{base} = r\sqrt{3}$

$h = ?\,;\ \sin\alpha = \dfrac{\text{cat. op. } \alpha}{\text{hip.}}$



$\sin 60° = \dfrac{h}{r\sqrt{3}}$

$\Leftrightarrow \dfrac{\sqrt{3}}{2} \times r\sqrt{3} = h$

$\Leftrightarrow h = \dfrac{3}{2}r$

$A_{\triangle\,[ABC]} = \dfrac{r\sqrt{3} \times \frac{3}{2}r}{2}$

$\Leftrightarrow A_{\triangle\,[ABC]} = \dfrac{3\,r^2\sqrt{3}}{4}$

$A_{\odot} = \pi r^2$

Medida da área da parte colorida $= A_{\odot} - A_{\triangle\,[ABC]} =$

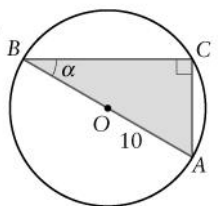
$= \pi r^2 - \dfrac{3\,r^2\sqrt{3}}{4} = \left(\pi - \dfrac{3\sqrt{3}}{4}\right) r^2$

$\dfrac{\text{Medida área da parte colorida}}{\text{Medida da área do } \triangle\,[ABC]} = \dfrac{\left(\pi - \frac{3\sqrt{3}}{4}\right)\cancel{r^2}}{\frac{3\,\cancel{r^2}\sqrt{3}}{4}} =$

$= \dfrac{\frac{4\pi}{4} - \frac{3\sqrt{3}}{4}}{\frac{3\sqrt{3}}{4}} = \dfrac{(4\pi - 3\sqrt{3}) \times \cancel{4}}{3\sqrt{3} \times \cancel{4}} =$

$= \dfrac{4\pi - 3\sqrt{3}}{3\sqrt{3}} = \dfrac{4\sqrt{3}\pi - 9}{9}$ c. q. d.

10.



$\overline{AB} = 10$ m

$\alpha = A\hat{B}C$

10.1 $[ABC]$ é triângulo rectângulo.

Sabemos que um ângulo inscrito numa circunferência tem amplitude igual a metade da amplitude do arco que o contém. Como $\overset{\frown}{AB} = 180°$, porque é metade do círculo, logo $A\hat{C}B = \dfrac{180°}{2} = 90°$.

10.2 $A(\alpha) = 10 \sin\alpha \cos\alpha \longrightarrow$ Área do triângulo

$A_{\triangle\,[ABC]} = \dfrac{b \times h}{2}$; $\ b = \text{base} = \overline{BC}$; $\ h = \text{altura} = \overline{AC}$

$\cos\alpha = \dfrac{\text{cat. adj. } \alpha}{\text{hip.}} \Leftrightarrow \cos\alpha = \dfrac{\overline{BC}}{10}$

$\Leftrightarrow \overline{BC} = 10 \cos \alpha$

$\sin \alpha = \dfrac{\text{cat. op. } \alpha}{\text{hip.}} \Leftrightarrow \sin \alpha = \dfrac{\overline{AC}}{10}$

$\Leftrightarrow \overline{AC} = 10 \sin \alpha$

$A_{\triangle [ABC]} = \dfrac{10 \cos \alpha \; 10 \sin \alpha}{2}$

$\Leftrightarrow A_{\triangle [ABC]} = 50 \cos \alpha \sin \alpha$ c. q. d.

10.3 a) $\alpha = \dfrac{\pi}{3}$

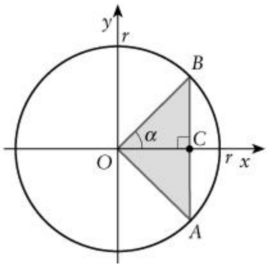
$A_{\triangle [ABC]} = 50 \cos \alpha \sin \alpha$

$= 50 \cos \dfrac{\pi}{3} \sin \dfrac{\pi}{3}$

$= 50 \times \dfrac{\sqrt{3}}{2} \times \dfrac{1}{2} = \dfrac{50\sqrt{3}}{4} = 12{,}5\sqrt{3}$

b) $\alpha = \dfrac{\pi}{2}$. Com este ângulo $\left(\dfrac{\pi}{2}\right)$ não é possível construir o triângulo.

11.



$A_{\triangle [AOB]} = A = r^2 \sin \alpha \cos \alpha$

$A_{\triangle [AOB]} = 2\, A_{\triangle [OCB]}$

$A_{\triangle [OCB]} = \dfrac{\overline{OC} \times \overline{CB}}{2}$

$\cos \alpha = \dfrac{\text{cat. adj. } \alpha}{\text{hip.}} \Leftrightarrow \cos \alpha = \dfrac{\overline{OC}}{r} \Leftrightarrow$

$\Leftrightarrow \overline{OC} = r \cos \alpha$

$\sin \alpha = \dfrac{\text{cat. op. } \alpha}{\text{hip.}} \Leftrightarrow \sin \alpha = \dfrac{\overline{CB}}{r} \Leftrightarrow$

$\Leftrightarrow \overline{CB} = r \sin \alpha$

$A_{\triangle [OCB]} = \dfrac{r \cos \alpha \times r \sin \alpha}{2} = \dfrac{r^2 \cos \alpha \sin \alpha}{2}$

$A_{\triangle [AOB]} = 2\, A_{\triangle [OCB]} = \dfrac{\cancel{2} \times r^2 \cos \alpha \sin \alpha}{\cancel{2}} =$

$= r^2 \cos \alpha \sin \alpha$ c. q. d.

12. $A_{\text{Jardim}} = A_{\text{Sector circular}} - A_{\triangle}$

$43° = ?$ (rad.)

π rad — 180°

x — 43°

$x = \dfrac{43 \times \pi}{180}$ rad

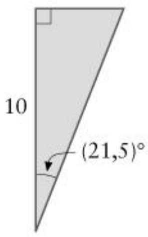
$A_{\text{Sector circ.}} = \dfrac{r^2 \alpha}{2} \Leftrightarrow A_{\text{sector circ.}} = \dfrac{25^2 \times \dfrac{43\pi}{180}}{2}$

$\Leftrightarrow A_{\text{Sector circ.}} = \dfrac{26875\pi}{360} = \dfrac{5375\pi}{72}$

$A_{\triangle} = 2 \times A_{\triangledown}$

$A_{\triangle} = \dfrac{b \times h}{2}$

base b :



$\tan \alpha = \dfrac{\text{cat. op. } \alpha}{\text{cat. adj. } \alpha}$

$\tan 21{,}5° = \dfrac{b}{10} \Leftrightarrow b = 10 \tan 21{,}5°$

$A_{\triangledown} = \dfrac{10 \tan 21{,}5° \times 10}{2} \Leftrightarrow A_{\triangledown} = 50 \tan 21{,}5°$

$A_{\triangle} = 2 \times 50 \tan 21{,}5° \Leftrightarrow A_{\triangle} = 100 \tan 21{,}5°$

$A_{\text{Jardim}} = A_{\text{Sector circ.}} - A_{\triangle}$

$= \left(\dfrac{5375\pi}{72} - 100 \tan (21{,}5°)\right) \text{m}^2$