1.1 $\vec{a}\cdot\vec{b} = 5 \times 4 \times \cos(38°)$ **Pág. 161**

$= 20\cos(38°) \approx 15{,}76$;

1.2 $\vec{a}\cdot\vec{b} = 3 \times 4 \times \cos 90° = 0$;

1.3 $\vec{a}\cdot\vec{b} = 6{,}2 \times 8{,}3 \times \cos(132°)$

$= 51{,}46\cos(132°) \approx -34{,}43$;

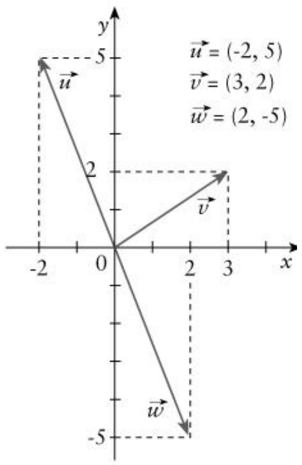
1.4 $\vec{a}\cdot\vec{b} = 3 \times 3{,}2 \times \cos\left(\frac{4\pi}{5}\text{ rad}\right)$

$= 9{,}6\cos\left(\frac{4\pi}{5}\text{ rad}\right) \approx -7{,}77$;

1.5 $\vec{a}\cdot\vec{b} = 4 \times 3 \times \cos\pi = -12$;

1.6 $\vec{a}\cdot\vec{b} = 6 \times 4 \times \cos(1\text{ rad}) \approx 12{,}97$.

2.1 a) **Pág. 164**



b) $\|\vec{u}\| = \sqrt{4+25} = \sqrt{29}$;

$\|\vec{v}\| = \sqrt{9+4} = \sqrt{13}$;

$\|\vec{w}\| = \sqrt{4+25} = \sqrt{29}$;

c) $\vec{u}\cdot\vec{v} = (-2\ ,\ 5)\cdot(3\ ,\ 2) = -6+10 = 4$;

$\vec{v}\cdot\vec{w} = (3\ ,\ 2)\cdot(2\ ,\ -5) = 6-10 = -4$;

$\vec{u}\cdot\vec{w} = (-2\ ,\ 5)\cdot(2\ ,\ -5) = -4-25 = -29$;

d$_1$) $\cos(\vec{u}\,\hat{,}\,\vec{v}) = \frac{4}{\sqrt{29}\times\sqrt{13}} = \frac{4}{\sqrt{377}}$

$\Rightarrow (\vec{u}\,\hat{,}\,\vec{v}) \approx (78{,}11)°$;

d$_2$) $\cos(\vec{v}\,\hat{,}\,\vec{w}) = \frac{-4}{\sqrt{13}\times\sqrt{29}} = -\frac{4}{\sqrt{377}}$

$\Rightarrow (\vec{v}\,\hat{,}\,\vec{w}) \approx (101{,}89)°$;

d$_3$) $\cos(\vec{u}\,\hat{,}\,\vec{w}) = \frac{-29}{\sqrt{29}\times\sqrt{29}} = -1$

$\Rightarrow (\vec{u}\,\hat{,}\,\vec{w}) \approx 180°$.

2.2 $A \curvearrowright (3\ ,\ 4)$; $B \curvearrowright (-2\ ,\ 1)$ e

$C \curvearrowright (-4\ ,\ -2)$;

2.2 a) $\overrightarrow{AB} = B - A = (-5\ ,\ -3)$; $\overrightarrow{BC} = C - B = (-2\ ,\ -3)$

$\overrightarrow{AC} = C - A = (-7\ ,\ -6)$; $\overrightarrow{BA} = -\overrightarrow{AB} = (5\ ,\ 3)$;

a$_1$) $\overrightarrow{AB}\cdot\overrightarrow{BC} = (-5\ ,\ -3)\cdot(-2\ ,\ -3) = 10+9 = 19$;

a$_2$) $\overrightarrow{AC}\cdot\overrightarrow{BA} = (-7\ ,\ -6)\cdot(5\ ,\ 3)$

$= -35-18 = -53$;

b) $\cos\left(\overrightarrow{AB}\,\hat{,}\,\overrightarrow{BC}\right) = \frac{\overrightarrow{AB}\cdot\overrightarrow{BC}}{\|\overrightarrow{AB}\|\cdot\|\overrightarrow{BC}\|}$

$= \frac{19}{\sqrt{25+9}\,\sqrt{4+9}} = \frac{19}{\sqrt{442}}$

$\Rightarrow \left(\overrightarrow{AB}\,\hat{,}\,\overrightarrow{BC}\right) \approx 25{,}35°$.

2.3 $\vec{u} = (4\ ,\ 2)$

a) Por exemplo: $\vec{a} = (2\ ,\ 4)$, $\vec{b} = (-2\ ,\ 4)$ e $\vec{c} = (4\ ,\ -8)$;

b) Seja $\vec{n} = (a\ ,\ b)$ e $\vec{n} \perp \vec{u}$

$\vec{n} \perp \vec{u} \Leftrightarrow (a\ ,\ b)\cdot(4\ ,\ 2) = 0$

$\Leftrightarrow 4a + 2b = 0 \Leftrightarrow 2a + b = 0 \Leftrightarrow b = -2a$

$\vec{n} = (a\ ,\ -2a)$, para $a \in \{0\}$, define a família de vectores perpendiculares a $\vec{u}$.

3. $A \curvearrowright (4\ ,\ 3)$; $B \curvearrowright (-2\ ,\ 1)$ **Pág. 166**

3.1 $\overrightarrow{AB} = B - A = (-6\ ,\ -2)$

$(-6\ ,\ -2)$;

3.2 a) Seja M o ponto médio de $[AB]$

$M \curvearrowright (1\ ,\ 2)$

Seja $P(x\ ,\ y)$ um ponto da mediatriz de

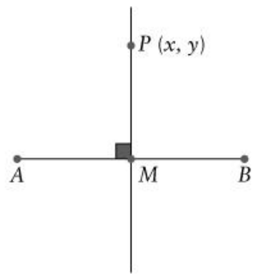
$\overrightarrow{MP}\cdot\overrightarrow{AB} = 0$

$\Leftrightarrow (x-1\ ,\ y-2)\cdot(-6\ ,\ -2) = 0$

$\Leftrightarrow -6(x-1) - 2(y-2) = 0$

$\Leftrightarrow -6x + 6 - 2y + 4 = 0$

$\Leftrightarrow 3x + y - 5 = 0$;



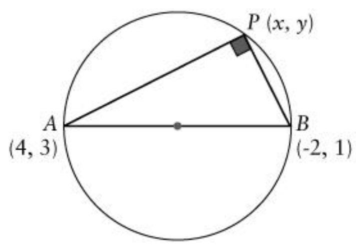
b) $\overrightarrow{AP}\cdot\overrightarrow{BP} = 0$

$\Leftrightarrow (x-4\ ,\ y-3)\cdot(x+2\ ,\ y-1) = 0$

$\Leftrightarrow (x-4)(x+2) + (y-3)(y-1) = 0$

$\Leftrightarrow x^2 + 2x - 4x - 8 + y^2 - y - 3y + 3 = 0$

$\Leftrightarrow x^2 + y^2 - 2x - 4y - 5 = 0$;

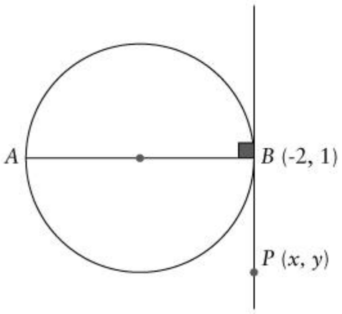


c) $\overrightarrow{BP}\cdot\overrightarrow{AB} = 0$

$\Leftrightarrow (x+2\ ,\ y-1)\cdot(-6,\ -2) = 0$

$\Leftrightarrow -6x - 12 - 2y + 2 = 0$

$\Leftrightarrow 3x + y + 5 = 0$.



3.3 $A \curvearrowright (1\ ,\ 0)\ ;\ B \curvearrowright (-1\ ,\ 4)$

a) Ponto médio de $[AB]$: $M \curvearrowright (0\ ,\ 2)$

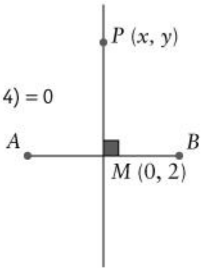
$\overrightarrow{AB} = (-2\ ,\ 4)$

$\overrightarrow{MP} \cdot \overrightarrow{AB} = 0$

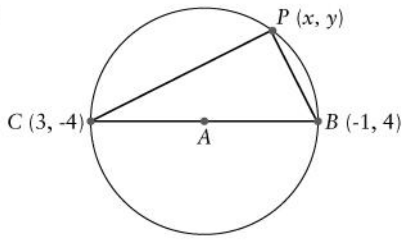
$\Leftrightarrow\ (x\ ,\ y-2) \cdot (-2\ ,\ 4) = 0$

$\Leftrightarrow\ -2x + 4y - 8 = 0$

$\Leftrightarrow\ x - 2y + 4 = 0\ ;$



b)



$[CB]$ é um diâmetro da circunferência sendo

$C = A + \overrightarrow{BA} = (1\ ,\ 0) + (2\ ,\ -4) = (3\ ,\ -4)$

$\overrightarrow{CP} \cdot \overrightarrow{BP} = 0$

$\Leftrightarrow\ (x-3\ ,\ y+4) \cdot (x+1\ ,\ y-4) = 0$

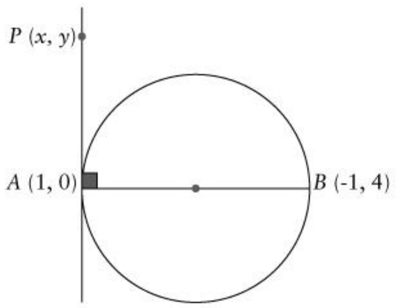
$\Leftrightarrow\ x^2 + y^2 - 2x - 19 = 0$

ou circunferência de centro $A \curvearrowright (1\ ,\ 0)$

e raio $\|\overrightarrow{AB}\| = \sqrt{4+16} = \sqrt{20}$:

$(x-1)^2 + y^2 = 20\ \Leftrightarrow\ x^2 + y^2 - 2x - 19 = 0\ ;$

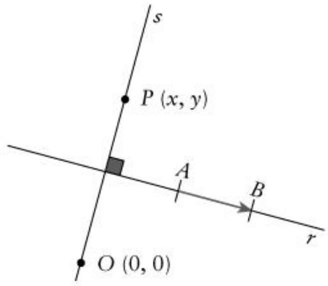
c)



$\overrightarrow{AP} \cdot \overrightarrow{AB} = 0\ \Leftrightarrow\ (x-1\ ,\ y) \cdot (-2\ ,\ 4) = 0$

$\Leftrightarrow\ -2x + 2 + 4y = 0\ \Leftrightarrow\ x - 2y - 1 = 0\ ;$

d)



Recta s :

$\overrightarrow{OP} \cdot \overrightarrow{AB} = 0\ \Leftrightarrow\ (x\ ,\ y) \cdot (-2\ ,\ 4) = 0$

$\Leftrightarrow\ -2x + 4y = 0\ \Leftrightarrow\ x - 2y = 0\ .$

4.1 a) $r:\ (0\ ,\ 3) + k\,(1\ ,\ 3)\ ,\ k \in \mathbb{R}\ ;$ **Pág. 167**

$s:\ (x\ ,\ y) = (-1\ ,\ 5) + k\,(-1\ ,\ 2)\ ,\ k \in \mathbb{R}$

$\vec{r} = (1\ ,\ 3)\ ;\ \ \vec{s} = (-1\ ,\ 2)$

$$\cos\alpha = \frac{|(1\ ,\ 3)\cdot(-1\ ,\ 2)|}{\sqrt{1+9}\times\sqrt{1+4}} = \frac{|-1+6|}{\sqrt{10}\ \sqrt{5}}$$

$$= \frac{5}{5\sqrt{2}} = \frac{\sqrt{2}}{2}\ \Longrightarrow\ \alpha = 45°\ ;$$

b) $r:\ y = 2x + 1\ ;\ \ s = -3x$

$\vec{r} = (1\ ,\ 2)\ ;\ \ \vec{s} = (1\ ,\ -3)$

$$\cos\alpha = \frac{|(1\ ,\ 2)\cdot(1\ ,\ -3)|}{\sqrt{1+4}\times\sqrt{1+9}} = \frac{|1-6|}{\sqrt{5}\ \sqrt{10}}$$

$$= \frac{5}{5\sqrt{2}} = \frac{\sqrt{2}}{2}\ \Longrightarrow\ \alpha = 45°\ ;$$

c) $r:\ x + y - 1 = 0\ ;\ \ s:\ y = x + 2$

$\vec{r} = (1\ ,\ -1)\ ;\ \ \vec{s} = (1\ ,\ 1)$

$$\cos\alpha = \frac{|(1\ ,\ -1)\cdot(1\ ,\ 1)|}{\sqrt{1+1}\times\sqrt{1+1}} = 0$$

$\Longrightarrow\ \alpha = 90°\ ;$

d) $r:\ y = 2x + 3\ ;\ \ s:\ y = 2x - 3$

$r\ //\ s\ ;\ \ (r\ \hat{,}\ s) = 0°\ .$

4.2 $A \curvearrowright (4\ ,\ 1)\ ;\ B \curvearrowright (0\ ,\ 2)\ ;\ C \curvearrowright (-2\ ,\ -2)$

$\overrightarrow{BC} = (-2\ ,\ -4)\ ;\ \ \overrightarrow{BA} = (4\ ,\ -1)\ ;\ \ \overrightarrow{CA} = (6\ ,\ 3)$

$$\cos\left(\overrightarrow{BC}\ \hat{,}\ \overrightarrow{BA}\right) = \frac{(-2\ ,\ -4)\cdot(4\ ,\ -1)}{\sqrt{4+16}\times\sqrt{16+1}}$$

$$= \frac{-8+4}{\sqrt{20}\ \sqrt{17}} = \frac{-4}{\sqrt{340}}$$

$$\Longrightarrow\ \left(\overrightarrow{BC}\ \hat{,}\ \overrightarrow{BA}\right) \approx (102{,}529)°$$

$$\cos\left(\overrightarrow{CB}\ \hat{,}\ \overrightarrow{CA}\right) = \frac{(2\ ,\ 4)\cdot(6\ ,\ 3)}{\sqrt{20}\ \sqrt{36+9}}$$

$$= \frac{12+12}{\sqrt{20}\ \sqrt{45}} = \frac{24}{30} = \frac{4}{5}$$

$$\Longrightarrow\ \left(\overrightarrow{CB}\ \hat{,}\ \overrightarrow{CA}\right) \approx (36{,}870)°$$

$\left(\overrightarrow{AC}\ \hat{,}\ \overrightarrow{AB}\right) \approx 180° - 102{,}529° - 36{,}870° = (40{,}601)°$

$\hat{A} \approx (40{,}601)°\ ;\ \ \hat{B} \approx (102{,}529)°\ ;\ \ \hat{C} \approx (36{,}870)°\ .$

4.3 $r:\ y = x;\quad \vec{r} = (1\,,\ 1)$

$s:\ (\sqrt{3}-1)\,x + (\sqrt{3}+1)\,y = \sqrt{3};$

$\vec{s} = (\sqrt{3}+1\,,\ -\sqrt{3}+1)$

$\vec{r}\cdot\vec{s} = (\sqrt{3}+1) + (-\sqrt{3}+1) = 2$

$\|\vec{r}\| = \sqrt{2}$

$\|\vec{s}\| = \sqrt{(\sqrt{3}+1)^2 + (-\sqrt{3}+1)^2} =$

$= \sqrt{3+1+2\sqrt{3}+3+1-2\sqrt{3}} = 2\sqrt{2}$

$\cos\alpha = \dfrac{|\vec{r}\cdot\vec{s}|}{\|\vec{r}\|\,\|\vec{s}\|} = \dfrac{2}{\sqrt{2}\times 2\sqrt{2}} = \dfrac{1}{2}$

$\Longrightarrow\ \alpha = 60°.$

5.1 a) . $a:\ y = -3x + 1$ **Pág. 168**

$\alpha = \tan^{-1}(-3) + 180° \approx (108{,}43)°;$

b) . $b:\ y = 2x + \dfrac{1}{2}$

$\alpha = \tan^{-1}(2) \approx 63{,}43°;$

c) . $c:\ x + 2y - 2 = 0 \iff 2y = -x + 2$

$\iff y = -\dfrac{1}{2}x + 1$

$\alpha = \tan^{-1}\left(-\dfrac{1}{2}\right) + 180 \approx 153{,}43°;$

d) . $d:\ y = -2x$

$\alpha = \tan^{-1}(-2) + 180 \approx 116{,}57°;$

e) . $e:\ y = -50$

$\alpha = \tan^{-1}(0) = 0°;$

f) . $f:\ x = 1800$

O declive não está definido; $\alpha = 90°$.

5.2 $A \curvearrowright (-1\,,\ 5)$

a) . $m = \tan\left(\dfrac{\pi}{3}\text{ rad}\right) = \sqrt{3}$

$y - 5 = \sqrt{3}\,(x+1) \iff y = \sqrt{3}x + \sqrt{3} + 5$

$y = \sqrt{3}x + \sqrt{3} + 5;$

b) . $m = \tan(1\text{ rad}) \approx 1{,}56$ (2 c. d.)

$y - 5 = 1{,}56\,(x+1)$

$\iff y = 1{,}56x + 6{,}56$ (2 c. d.);

c) . $m = \tan(45°) = 1$

$y - 5 = 1\,(x+1) \iff y = x + 6;$

d) . $m = \tan(15°) \approx 0{,}27$

$y - 5 = 0{,}27\,(x+1)$

$\iff y = 0{,}27x + 5{,}27$ (2 c. d.)

6.1 a) -1 **Pág. 169**

b) $\dfrac{-1}{-2} = \dfrac{1}{2}$

c) $\dfrac{-1}{-5} = \dfrac{1}{5}$

d) $\dfrac{-1}{\frac{1}{2}} = -2$

e) $\dfrac{-1}{\sqrt{2}} = \dfrac{-1\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = -\dfrac{\sqrt{2}}{2}$

f) $\dfrac{-1}{\sqrt{3}-1} = \dfrac{-1(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \dfrac{-\sqrt{3}-1}{3-1} = -\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}$

g) $\dfrac{-1}{\sqrt{2}-\sqrt{3}} = \dfrac{-1(\sqrt{2}+\sqrt{3})}{(\sqrt{2}-\sqrt{3})(\sqrt{2}+\sqrt{3})}$

$= \dfrac{-\sqrt{2}-\sqrt{3}}{2-3} = \sqrt{2}+\sqrt{3}$

6.2 $y = \dfrac{1}{2}x;\quad y = \dfrac{1}{2}x + 1;\quad y = \dfrac{1}{2}x + 2;$

$y = \dfrac{1}{2}x - 1;\quad y = \dfrac{1}{2}x - 2$ (por exemplo).

6.3 $A \curvearrowright (1\,,\ 5);\quad m = 2$

$y - 5 = 2\,(x-1) \iff y = 2x + 3.$

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1. $\overline{AB} = 3;\ \overline{AC} = 2$ **Pág. 170**

$\overrightarrow{BC} = C - B = (1,4) - (4,2)$

$= (-3,2)$

$\|\overrightarrow{BC}\| = \sqrt{(-3)^2 + 2^2} = \sqrt{13}$

O perímetro, P, do triângulo é:

$P = 3 + 2 + \sqrt{13} \Leftrightarrow P = 5 + \sqrt{13}$

Resposta: (C).

2. O ponto $(0\,,-1)$ é um ponto de r.

$s:\ y = 2x + 5$

Então:

$d = \dfrac{|-1 - 2\times 0 - (+5)|}{\sqrt{1+2^2}} = \dfrac{6}{\sqrt{5}} = \dfrac{6\sqrt{5}}{5}$

Nota: Por lapso, nenhuma das alternativas está correcta.

3. (A) Esta afirmação é falsa.

As rectas não são paralelas porque têm declives diferentes.

(B) Esta afirmação é falsa.

O ângulo das rectas a e b é agudo e o ângulo dos vectores $\vec{u}$ e $\vec{v}$ é recto.

(C) Esta afirmação é falsa.

Se α é a inclinação de uma recta r e $\alpha \neq 90°$, o declive de r é $\tan(\alpha)$.

(D) É verdade, porque $\dfrac{1}{2} = -\dfrac{1}{-2}$.

Resposta: (D).

4. Sendo $P(x\,,\,y)$ um ponto genérico das rectas p e q, tem-se:

$d(P\,,r) = d(P\,,s) \Leftrightarrow$

$\Leftrightarrow \dfrac{|3x+4y-2|}{\sqrt{3^2+4^2}} = \dfrac{|4x+3y+5|}{\sqrt{4^2+3^2}}$

$\Leftrightarrow \dfrac{|3x+4y-2|}{5} = \dfrac{|4x+3y+5|}{5}$

$\Leftrightarrow 3x+4y-2 = 4x+3y+5 \vee 3x+4y-2 = -(4x+3y+5)$

$\Leftrightarrow x - y + 7 = 0 \vee 7x + 7y + 3 = 0$

Logo, $x - y + 7 = 0 \vee 7x + 7y + 3 = 0$ são as equações das rectas p e q.

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5. $L(5\,,\ 5);\quad U(1\,,\ 1);\quad A(5\,,\ 0)$ **Pág. 171**

$\overrightarrow{UL} = (4\,,\ 4);\quad \overrightarrow{UA} = (4\,,\ -1);\quad \overrightarrow{LA} = (0\,,\ -5)$

• $\cos\left(\overrightarrow{UL}\ \hat{,}\ \overrightarrow{UA}\right) = \dfrac{\overrightarrow{UL}\cdot\overrightarrow{UA}}{\|\overrightarrow{UL}\|\,\|\overrightarrow{UA}\|}$

$= \dfrac{(4\ ,\ 4)\cdot(4\ ,\ -1)}{\sqrt{16+16}\times\sqrt{16+1}} = \dfrac{16-4}{\sqrt{32}\ \sqrt{17}}$

$= \dfrac{12}{\sqrt{544}}$

$\left(\overrightarrow{UL}\ \hat{,}\ \overrightarrow{UA}\right) = \cos^{-1}\left(\dfrac{12}{\sqrt{544}}\right) \approx 59°$

- $\cos\left(\overrightarrow{LU}\ \hat{,}\ \overrightarrow{LA}\right) = \dfrac{\overrightarrow{LU}\cdot\overrightarrow{LA}}{\|\overrightarrow{LU}\|\ \|\overrightarrow{LA}\|}$

$= \dfrac{(-4\ ,\ 4)\cdot(0\ ,\ -5)}{\sqrt{16+16}\cdot 5} = \dfrac{20}{4\sqrt{2}\times 5} = \dfrac{\sqrt{2}}{2}$

$\left(\overrightarrow{LU}\ \hat{,}\ \overrightarrow{LA}\right) = 45°$

- $\left(\overrightarrow{AU}\ \hat{,}\ \overrightarrow{AL}\right) \approx 180 - 59° - 45° = 76°$

$\hat{U} \approx 59°\ ;\quad \hat{L} = 45°\ ;\quad \hat{A} \approx 76°\ .$

6. $\|\vec{u}\| = \sqrt{5}\ ;\quad \|\vec{v}\| = 1\ ;\quad (\vec{u}\ \hat{,}\ \vec{v}) = 45°\ ;$

$\vec{a} = \vec{u} + \vec{v}\ ;\quad \vec{b} = \vec{u} - \vec{v}$

$\cos(\vec{a}\ \hat{,}\ \vec{b}) = \dfrac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\ \|\vec{b}\|}$

$\vec{a}\cdot\vec{b} = (\vec{u}+\vec{v})\cdot(\vec{u}-\vec{v}) = \|\vec{u}\|^2 - \|\vec{v}\|^2$

$= 5 - 1 = 4$

$\|\vec{a}\| = \sqrt{\vec{a}\cdot\vec{a}} = \sqrt{(\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v})}$

$= \sqrt{\|\vec{u}\|^2 + 2\vec{u}\cdot\vec{v} + \|\vec{v}\|^2}$

$= \sqrt{5 + 2\sqrt{5} - 1\cos 45° + 1} = \sqrt{6+\sqrt{10}}$

$\|\vec{b}\| = \sqrt{\vec{b}\cdot\vec{b}} = \sqrt{(\vec{u}-\vec{v})\cdot(\vec{u}-\vec{v})} =$

$= \sqrt{\|\vec{u}\|^2 - 2\vec{u}\cdot\vec{v} + \|\vec{v}\|^2} = \sqrt{6-\sqrt{10}}$

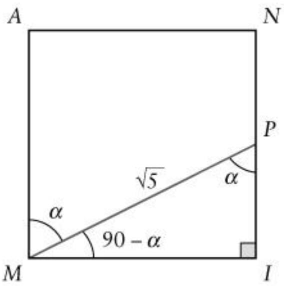
$= \dfrac{4}{\sqrt{6+\sqrt{10}}\cdot\sqrt{6-\sqrt{10}}}$

$= \dfrac{4}{\sqrt{(6+\sqrt{10})\ (6-\sqrt{10})}}$

$= \dfrac{4}{\sqrt{36-10}} = \dfrac{4}{\sqrt{26}}$

$(\vec{a}\ \hat{,}\ \vec{b}) = \cos^{-1}\left(\dfrac{4}{\sqrt{26}}\right) \approx (38{,}33)°\ .$

7.1



$\left(\overline{MI}\right)^2 + \left(\overline{IP}\right)^2 = \left(\overline{MP}\right)^2$

$\Leftrightarrow\ \overline{MI}^2 + \left(\dfrac{1}{2}\,\overline{MI}\right)^2 = 5 \ \Leftrightarrow\ \dfrac{5}{4}\left(\overline{MI}\right)^2 = 5$

$\Leftrightarrow\ \overline{MI}^2 = 4 \overset{\overline{MI}^2>0}{\Leftrightarrow} \overline{MI} = 2\ ;$

7.2 $\overrightarrow{MP} = \overrightarrow{MI} + \overrightarrow{IP} = \overrightarrow{MI} + \dfrac{1}{2}\overrightarrow{MA} \ = \overrightarrow{MI} - \dfrac{1}{2}\overrightarrow{AM}\ ;$

7.3 $\cos\alpha = \sin(90-\alpha) = \sin\left(\overrightarrow{MI}\ \hat{,}\ \overrightarrow{MP}\right) = \dfrac{\overline{IP}}{\sqrt{5}}$

$= \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}\ ;$

7.4 $\overrightarrow{AM}\cdot\overrightarrow{MP} = -\overrightarrow{MA}\cdot\overrightarrow{MP} = -\|\overrightarrow{MA}\|\times\|\overrightarrow{MP}\|\cos\alpha$

$= -2\times\sqrt{5}\times\dfrac{\sqrt{5}}{5} = -2$

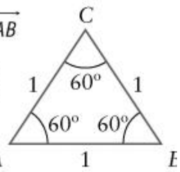
8. $\overrightarrow{AB}\cdot\overrightarrow{BC} + \overrightarrow{BC}\cdot\overrightarrow{CA} + \overrightarrow{CA}\cdot\overrightarrow{AB}$

$= -\overrightarrow{BA}\cdot\overrightarrow{BC} - \overrightarrow{CB}\cdot\overrightarrow{CA} - \overrightarrow{AC}\cdot\overrightarrow{AB}$

$= -1\times 1\times\cos 60° - 1\times 1\times$

$\times\cos 60° - 1\times 1\times\cos 60°$

$= -\dfrac{3}{2}\ .$



9. $\|\vec{u}+\vec{v}\|^2 = (\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v})$

$\|\vec{u}+\vec{v}\|^2 = \vec{u}\cdot\vec{u} + \vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{u} + \vec{v}\cdot\vec{v}$

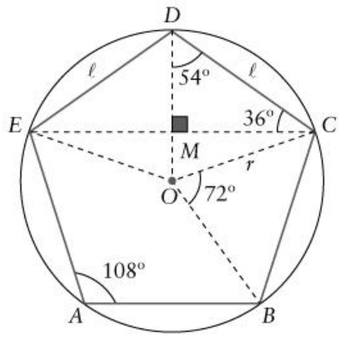
$\|\vec{u}+\vec{v}\|^2 = \|\vec{u}\|^2 + 2\vec{u}\cdot\vec{v} + \|\vec{v}\|^2$

Logo, $2\vec{u}\cdot\vec{v} = \|\vec{u}+\vec{v}\|^2 - \|\vec{u}\|^2 - \|\vec{v}\|^2$

10.1 $\overrightarrow{OC}\cdot\overrightarrow{OE} = \|\overrightarrow{OC}\|\times\|\overrightarrow{OE}\|\ \cos\left(\overrightarrow{OC}\ \hat{,}\ \overrightarrow{OE}\right)$

$= r\times r\cos(2\times 72°)$

$= r^2\cos(144)° \approx -0{,}81\ r^2$



10.2 $\overrightarrow{EC}\cdot\overrightarrow{CD} = -\overrightarrow{CE}\cdot\overrightarrow{CD}$

$= -\|\overrightarrow{CE}\|\times\|\overrightarrow{CD}\|\ \cos\left(\overrightarrow{CE}\ \hat{,}\ \overrightarrow{CD}\right)$

$= 2\ \ell\cos(36)°\times\ell\cos(36)°$

$\dfrac{\overline{CM}}{\ell} = \cos(36)°\ ;\quad \overline{CM} = \ell\cos(36)°\ ;\quad \overline{CE} = 2\ell\cos(36)°$

$= 2\ \ell^2\cos^2(36) \approx -1{,}31\ \ell^2$

10.3 $\overline{EC} = 12\text{ cm}$

10.3.1 $\overline{EC} = 2\ell\cos(36)°$

$12 = 2\ell\cos(36)° \ \Leftrightarrow\ \ell = \dfrac{6}{\cos(36)°}$

$\|\overrightarrow{ED}\| = \dfrac{6}{\cos(36)°}\text{ cm} \approx 7{,}42\text{ cm}\ ;$

10.3.2 $\overrightarrow{EC}\cdot\overrightarrow{ED} = \|\overrightarrow{EC}\|\times\|\overrightarrow{ED}\|\ \cos\left(\overrightarrow{EC}\ \hat{,}\ \overrightarrow{ED}\right)$

$= 12\times\dfrac{6}{\cos(36)°}\times\cos(36)°$

$= 72\ ;$

10.3.3 $\overrightarrow{EC}\cdot\overrightarrow{DC} = -\overrightarrow{CE}\cdot\left(-\overrightarrow{CD}\right) = \overrightarrow{CE}\cdot\overrightarrow{CD}$

$= \|\overrightarrow{CE}\|\times\|\overrightarrow{CD}\|\ \cos\left(\overrightarrow{CE}\ \hat{,}\ \overrightarrow{CD}\right)$

$= 12\times\dfrac{6}{\cos(36)°}\times\cos(36)°$

$= 72\ .$