Capítulo 5

1. Vectores no plano. Operações com vectores

1.1 a) $V_{\text{cubo}} = a^3$ **Pág. 122**

$a^3 = 27 \Leftrightarrow a = \sqrt[3]{27} \Leftrightarrow a = 3$,

então: $\|\overrightarrow{AB}\| = 3$;

b) $\|\overrightarrow{AC}\| = \sqrt{3^2 + 3^2} \Leftrightarrow \|\overrightarrow{AC}\| = \sqrt{18} \Leftrightarrow \|\overrightarrow{AC}\| = 3\sqrt{2}$;

c) $\|\overrightarrow{AG}\| = \sqrt{(3\sqrt{2})^2 + 3^2} \Leftrightarrow \|\overrightarrow{AG}\| = \sqrt{18 + 9} \Leftrightarrow$

$\Leftrightarrow \|\overrightarrow{AG}\| = \sqrt{27} \Leftrightarrow \|\overrightarrow{AG}\| = 3\sqrt{3}$;

d) Por exemplo: $\overrightarrow{DA}$ e $\overrightarrow{HE}$; **e)** D; **f)** G; **g)** B;

h) H; **i)** A; **j)** B; **k)** B.

2.1 a) $\overrightarrow{AB} + \overrightarrow{BI} = \overrightarrow{AI}$; **b)** $\overrightarrow{EF} + \overrightarrow{JI} = \overrightarrow{EC}$; **Pág. 124**

c) $\overrightarrow{EJ} + \overrightarrow{HC} = \overrightarrow{EG}$; **d)** $\overrightarrow{HF} + \overrightarrow{CI} = \overrightarrow{HJ}$;

e) $\overrightarrow{AC} + \overrightarrow{IF} = \overrightarrow{EC}$; **f)** $\overrightarrow{AH} + \overrightarrow{HI} = \overrightarrow{AI}$;

g) $\overrightarrow{AB} + \overrightarrow{BA} = \vec{0}$;

3.1 a) $\overrightarrow{CD} + 2\overrightarrow{QR} = \overrightarrow{CD} + \overrightarrow{QM} = \overrightarrow{CD} + \overrightarrow{DF} = \overrightarrow{CF}$; **Pág. 125**

b) $\overrightarrow{GI} - \overrightarrow{DJ} = \overrightarrow{GI} + \overrightarrow{JD} = \overrightarrow{GI} + \overrightarrow{IC} = \overrightarrow{GC}$;

c) $\overrightarrow{NR} + 2\overrightarrow{KJ} = \overrightarrow{NR} + \overrightarrow{KI} = \overrightarrow{NR} + \overrightarrow{RH} = \overrightarrow{NH}$;

d) $\frac{1}{2}\overrightarrow{HR} + \overrightarrow{KE} = \overrightarrow{HQ} + \overrightarrow{KE} = \overrightarrow{JK} + \overrightarrow{KE} = \overrightarrow{JE}$;

3.2 a) $3\vec{a} - \vec{b} = \overrightarrow{IG} - \overrightarrow{KI} = \overrightarrow{IG} + \overrightarrow{KI} = \overrightarrow{IJ}$;

b) $2(\vec{a} + \vec{b}) = 2(\overrightarrow{OA} + \overrightarrow{OB}) = 2\overrightarrow{OH} = \overrightarrow{OC}$;

c) $3(\vec{b} - \vec{a}) = 3(\overrightarrow{OB} + \overrightarrow{AO}) = 3(\overrightarrow{OB} + \overrightarrow{OK}) = 3\overrightarrow{OI} = \overrightarrow{LF}$;

d) $-3(\vec{a} + \vec{b}) = -3(\overrightarrow{OA} + \overrightarrow{OB}) = -3\overrightarrow{OH} = 3\overrightarrow{HO} = \overrightarrow{CP}$;

e) $2\vec{a} + \vec{b} - 2\vec{b} - 3\vec{a} = -\vec{a} - \vec{b} = -(\vec{a} + \vec{b}) = -\overrightarrow{OH} = \overrightarrow{HO}$;

f) $-3(\vec{a} + \vec{b}) + 2(2\vec{a} - \vec{b}) = -3\vec{a} + 3\vec{b} + 4\vec{a} - 2\vec{b} = \vec{a} + \vec{b} = \overrightarrow{OH}$.

3.3 a₁) $-2\overrightarrow{BM} = \overrightarrow{AB}$;

a₂) $\frac{1}{2}\overrightarrow{FE} + \overrightarrow{FG} = \overrightarrow{FO} + \overrightarrow{FG} = \overrightarrow{FG} + \overrightarrow{GP} = \overrightarrow{FP}$;

a₃) $\frac{1}{2}\overrightarrow{HG} = \overrightarrow{HP}$;

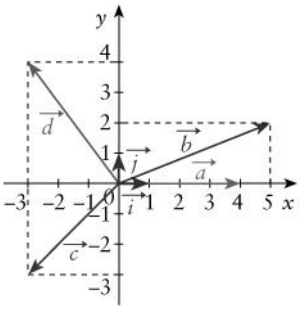
a₄) $2\overrightarrow{NB} + \overrightarrow{BE} = \overrightarrow{CB} + \overrightarrow{BE} = \overrightarrow{CE}$.

b) $\overrightarrow{AM}$ é colinear com $\overrightarrow{GP}$ pois $\overrightarrow{AM} = -\overrightarrow{GP}$;

$\overrightarrow{AM}$ é colinear com $\overrightarrow{EF}$ pois $\overrightarrow{AM} = 0{,}5\overrightarrow{EF}$;

$\overrightarrow{AM}$ é colinear com $\overrightarrow{CD}$ pois $\overrightarrow{AM} = -0{,}5\overrightarrow{CD}$.

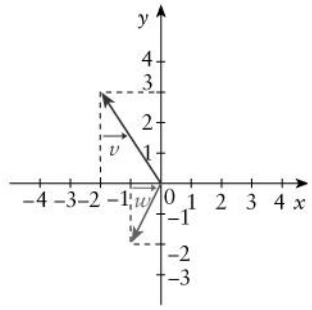
4.1 **Pág. 127**



4.2

Vector	Componentes	Coordenadas
$\vec{a}$	$4\vec{i}$ e $0\vec{j}$	$\vec{a}(4, 0)$
$\vec{b}$	$5\vec{i}$ e $2\vec{j}$	$\vec{b}(5, 2)$
$\vec{c}$	$-3\vec{i}$ e $-3\vec{j}$	$\vec{c}(-3, -3)$
$\vec{d}$	$-3\vec{i}$ e $4\vec{j}$	$\vec{d}(-3, 4)$

4.3 a) e b) Por exemplo:



5.1 a) $\overrightarrow{MN} = N - M \Leftrightarrow \overrightarrow{MN} = (0, -3) - (1, 2)$ **Pág. 128**

$\Leftrightarrow \overrightarrow{MN} = (0 - 1, -3 - 2) \Leftrightarrow \overrightarrow{MN} = (-1, -5)$;

b) $\overrightarrow{NM} = M - N \Leftrightarrow \overrightarrow{NM} = (1, 2) - (0, -3) \Leftrightarrow$

$\Leftrightarrow \overrightarrow{NM} = (1 - 0, 2 - (-3)) \Leftrightarrow \overrightarrow{NM} = (1, 5)$;

c) $\overrightarrow{NP} = P - N \Leftrightarrow \overrightarrow{NP} = (3, 0) - (0, -3) \Leftrightarrow$

$\Leftrightarrow \overrightarrow{NP} = (3 - 0, 0 - (-3)) \Leftrightarrow \overrightarrow{NP} = (3, 3)$;

5.2 a) $M + \overrightarrow{NM} = (1, 2) + (1, 5) \Leftrightarrow M + \overrightarrow{NM} = (1 + 1, 2 + 5)$

$\Leftrightarrow M + \overrightarrow{NM} = (2, 7)$;

b) $N + \overrightarrow{MN} = (0, -3) + (-1, -5) \Leftrightarrow$

$\Leftrightarrow N + \overrightarrow{MN} = (0 + (-1), -3 + (-5)) \Leftrightarrow$

$\Leftrightarrow N + \overrightarrow{MN} = (-1, -8)$;

c) $N + \overrightarrow{MP} = N + (P - M) \Leftrightarrow$

$\Leftrightarrow N + \overrightarrow{MP} = (0, -3) + ((3, 0) - (1, 2))$

$\Leftrightarrow N + \overrightarrow{MP} = (0, -3) + (3 - 1, 0 - 2) \Leftrightarrow$

$\Leftrightarrow N + \overrightarrow{MP} = (0, -3) + (2, -2) \Leftrightarrow$

$\Leftrightarrow N + \overrightarrow{MP} = (0 + 2, -3 + (-2)) \Leftrightarrow$

$\Leftrightarrow N + \overrightarrow{MP} = (2, -5)$;

---

6. a) $\vec{a} + \vec{b} = (1, -1) + (-2, 3) \Leftrightarrow$ **Pág. 129**

$\Leftrightarrow \vec{a} + \vec{b} = (1 + (-2), -1 + 3) \Leftrightarrow \vec{a} + \vec{b} = (-1, 2)$;

b) $\vec{b} + \vec{c} = (-2, 3) + (-10, 5) \Leftrightarrow$

$\Leftrightarrow \vec{b} + \vec{c} = (-2 + (-10), 3 + 5) \Leftrightarrow \vec{b} + \vec{c} = (-12, 8)$;

c) $(\vec{a} + \vec{c}) + \vec{b} = ((1, -1) + (-10, 5)) + (-2, 3) \Leftrightarrow$

$\Leftrightarrow (\vec{a} + \vec{c}) + \vec{b} = (1 + (-10), -1 + 5) + (-2, 3) \Leftrightarrow$

$\Leftrightarrow (\vec{a} + \vec{c}) + \vec{b} = (-9, 4) + (-2, 3) \Leftrightarrow$

$\Leftrightarrow (\vec{a} + \vec{c}) + \vec{b} = (-9 + (-2), 4 + 3) \Leftrightarrow$

$\Leftrightarrow (\vec{a} + \vec{c}) + \vec{b} = (-11, 7)$;

d) $(\vec{a} + \vec{0}) + \vec{b} = ((1, -1) + (0, 0)) + (-2, 3) \Leftrightarrow$

$\Leftrightarrow (\vec{a} + \vec{0}) + \vec{b} = (1, -1) + (-2, 3) \Leftrightarrow$

$\Leftrightarrow (\vec{a} + \vec{0}) + \vec{b} = (-1, 2)$;

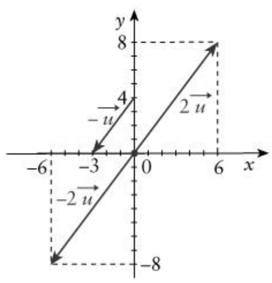
e) $\vec{a} + \vec{b} + \vec{c} + \vec{c} = \vec{a} + \vec{b} + (-10, 5) + (-10, 5) \Leftrightarrow$

$\Leftrightarrow \vec{a} + \vec{b} + \vec{c} + \vec{c} = (-1, 2) + (-10, 5) + (-10, 5) \Leftrightarrow$

$\Leftrightarrow \vec{a} + \vec{b} + \vec{c} + \vec{c} = (-1 - 10 - 10, 2 + 5 + 5) \Leftrightarrow$

$\Leftrightarrow \vec{a} + \vec{b} + \vec{c} + \vec{c} = (-21, 12)$;

7.1 a), b) e c) Por exemplo: **Pág. 130**



7.2 **a)** $\vec{a}-3\vec{a}=-2\vec{a} \Leftrightarrow \vec{a}-3\vec{a}=-2\,(-10\,,\ 5) \Leftrightarrow$
$\Leftrightarrow \vec{a}-3\vec{a}=(20\,,\ -10)\,;$

b) $\vec{a}-3\vec{a}+5\vec{a}=3\vec{a} \Leftrightarrow \vec{a}-3\vec{a}+5\vec{a}=3\,(-10\,,\ 5) \Leftrightarrow$
$\Leftrightarrow \vec{a}-3\vec{a}+5\vec{a}=(-30\,,\ 15)\,;$

c) $2(-3\vec{a})=-6\vec{a} \Leftrightarrow 2(-3\vec{a})=-6(-10\,,\ 5) \Leftrightarrow$
$\Leftrightarrow 2(-3\vec{a})=(60\,,\ -30)\,;$

d) $6\vec{a}-7\vec{a}+\vec{b}-2\vec{b}=-\vec{a}-\vec{b} \Leftrightarrow$
$\Leftrightarrow 6\vec{a}-7\vec{a}+\vec{b}-2\vec{b}=-(-10\,,\ 5)-(-8\,,\ 0) \Leftrightarrow$
$\Leftrightarrow 6\vec{a}-7\vec{a}+\vec{b}-2\vec{b}=(10\,,\ -5)+(8\,,\ 0) \Leftrightarrow$
$\Leftrightarrow 6\vec{a}-7\vec{a}+\vec{b}-2\vec{b}=(18\,,\ -5)\,;$

e) $-\frac{1}{2}\vec{a}+8\vec{a}-\frac{1}{2}\vec{b}+\frac{3}{4}\vec{b}=-\frac{1}{2}\vec{a}+\frac{16}{2}\vec{a}-\frac{2}{4}\vec{b}+\frac{3}{4}\vec{b}=$
$=\frac{15}{2}\vec{a}+\frac{1}{4}\vec{b}=\frac{15}{2}(-10\,,\ 5)+\frac{1}{4}(-8\,,\ 0)=$
$=\left(-\frac{150}{2}\,,\ \frac{75}{2}\right)+\left(-\frac{8}{4}\,,\ 0\right)=(-75\,;\ 37{,}5)+(-2\,,\ 0)=$
$=(-75+(-2)\,;\ 37{,}5)=(-77\,;\ 37{,}5)\,;$

f) $-\vec{a}+\frac{1}{2}\vec{a}+2(-\vec{b}+3\vec{a})=-\vec{a}+\frac{1}{2}\vec{a}-2\vec{b}+6\vec{a}=$
$=5\vec{a}+\frac{1}{2}\vec{a}-2\vec{b}=\frac{10}{2}\vec{a}+\frac{1}{2}\vec{a}-2\vec{b}=\frac{11}{2}\vec{a}-2\vec{b}=$
$=\frac{11}{2}(-10\,,\ 5)-2\,(-8\,,\ 0)=$
$=\left(-\frac{110}{2}\,,\ \frac{55}{2}\right)-(-16\,,\ 0)=\left(-55\,,\ \frac{55}{2}\right)+(16\,,\ 0)=$
$=\left(-39\,,\ \frac{55}{2}\right).$

8.1 **a)** $\|\vec{u}\|=\sqrt{(-10)^2+5^2} \Leftrightarrow \|\vec{u}\|=\sqrt{100+25}$ **Pág. 131**
$\Leftrightarrow \|\vec{u}\|=\sqrt{125} \Leftrightarrow \|\vec{u}\|=5\sqrt{5}\,;$

b) $\|\vec{v}\|=\sqrt{\left(-\frac{1}{2}\right)^2+3^2} \Leftrightarrow \|\vec{v}\|=\sqrt{\frac{1}{4}+9} \Leftrightarrow$
$\Leftrightarrow \|\vec{v}\|=\sqrt{\frac{37}{4}} \Leftrightarrow \|\vec{v}\|=\frac{\sqrt{37}}{2}.$

c) $\overrightarrow{AB}=B-A=(-2\,,\ -3)-(-1\,,\ 5)=(-1\,,\ -8)$
$\|\overrightarrow{AB}\|=\sqrt{(-1)^2+(-8)^2} \Leftrightarrow \|\overrightarrow{AB}\|=\sqrt{1+64} \Leftrightarrow$
$\Leftrightarrow \|\overrightarrow{AB}\|=\sqrt{65}\,;$

8.2 $\vec{a}$ e $\vec{b}$ são colineares se e só se existir um número real $k\neq 0$ tal que $\vec{a}=k\vec{b}$.
$\vec{a}=k\vec{b} \Leftrightarrow (2\,,\ 3)=k(-1\,,\ -1{,}5) \Leftrightarrow$
$\Leftrightarrow (2\,,\ 3)=(-k\,;\ -1{,}5k) \Leftrightarrow$
$\Leftrightarrow \begin{cases}-k=2\\-1{,}5k=3\end{cases} \Leftrightarrow \begin{cases}k=-2\\k=-2\end{cases} \Leftrightarrow k=-2$

Como $\vec{a}=-2\vec{b}$, conclui-se que $\vec{a}$ e $\vec{b}$ são colineares.

1.1 $A-\overrightarrow{HL}=A+\overrightarrow{LH}=A+\overrightarrow{AJ}=J$ **Pág. 132**

Resposta: **(C)**.

1.2 $\overrightarrow{AK}-\frac{1}{2}\overrightarrow{CI}=\overrightarrow{AK}-\overrightarrow{CH}=\overrightarrow{AK}+\overrightarrow{HC}=\overrightarrow{AK}+\overrightarrow{AE}=\overrightarrow{AL}$

Resposta: **(C)**.

2. $\overrightarrow{BC}=\overrightarrow{BA}+\overrightarrow{AC}$, regra do triângulo
$\overrightarrow{BC}=\overrightarrow{AC}+\overrightarrow{BA}$, propriedade comutativa
$\overrightarrow{BC}=\overrightarrow{AC}-\overrightarrow{AB}$

Resposta: **(B)**.

3. Sabemos que $\overrightarrow{CB}+\overrightarrow{BM}=\overrightarrow{CM} \Leftrightarrow \overrightarrow{BM}=\overrightarrow{CM}-\overrightarrow{CB} \Leftrightarrow$
$\Leftrightarrow \overrightarrow{BM}=\overrightarrow{CM}+\overrightarrow{BC}$ (1)

Por outro lado, $\overrightarrow{CB}+\overrightarrow{BA}=\overrightarrow{CA}$
$\vec{y}+\vec{x}=\overrightarrow{CA}$

Daqui resulta que: $\overrightarrow{CM}=\frac{1}{2}\overrightarrow{CA}=\frac{1}{2}\,(\vec{x}+\vec{y})$

Assim, substituindo, em (1) vem que:
$\overrightarrow{BM}=\frac{1}{2}\vec{x}+\frac{1}{2}\vec{y}-\vec{y} \Leftrightarrow \overrightarrow{BM}=\frac{1}{2}\vec{x}-\frac{1}{2}\vec{y} \Leftrightarrow \overrightarrow{BM}=\frac{1}{2}(\vec{x}-\vec{y})$

Resposta: **(A)**.

4. $\vec{u}$ e $\vec{v}$ são colineares se e só se existir um número real $k\neq 0$ tal que $\vec{v}=k\vec{u}$.
$\vec{v}=k\vec{u} \Leftrightarrow \vec{v}=k(-1\,,\ 2) \Leftrightarrow \vec{v}=(-k\,,\ 2k)$.

Por outro lado, temos que:
$\|\vec{v}\|=\frac{5}{4} \Leftrightarrow \sqrt{(-k)^2+(2k)^2}=\frac{5}{4} \Rightarrow k^2+4k^2=\frac{25}{16} \Leftrightarrow$
$\Leftrightarrow 5k^2=\frac{25}{16} \Leftrightarrow k^2=\frac{5}{16} \Leftrightarrow$
$\Leftrightarrow k=-\sqrt{\frac{5}{16}} \vee k=\sqrt{\frac{5}{16}} \Leftrightarrow$
$\Leftrightarrow k=-\frac{\sqrt{5}}{4} \vee k=\frac{\sqrt{5}}{4}$

Assim, vem:
$\vec{v}=-\frac{\sqrt{5}}{4}(-1\,,\ 2)$ ou $\vec{v}=\frac{\sqrt{5}}{4}(-1\,,\ 2)$
$\vec{v}=\left(\frac{\sqrt{5}}{4}\,,\ -\frac{\sqrt{5}}{2}\right)$ ou $\vec{v}=\left(-\frac{\sqrt{5}}{4}\,,\ \frac{\sqrt{5}}{2}\right)$

Resposta: **(B)**.

5.1 $\vec{w}=2\,(A-C)=2\,[(2\,,\ 0)-(0\,,\ -2)]=2\,(2\,,\ 2)=(4\,,\ 4)$
$\|\vec{w}\|=\sqrt{4^2+4^2} \Leftrightarrow \|\vec{w}\|=\sqrt{16+16} \Leftrightarrow \|\vec{w}\|=\sqrt{32} \Leftrightarrow$
$\Leftrightarrow \|\vec{w}\|=4\sqrt{2}$

Resposta: **(D)**.

5.2 $\overrightarrow{OA}-\frac{1}{2}\overrightarrow{BC}=(2\,,\ 0)-\frac{1}{2}(2\,,\ -4)$
$\overrightarrow{OA}-\frac{1}{2}\overrightarrow{BC}=(2\,,\ 0)-(1\,,\ -2)$
$\overrightarrow{OA}-\frac{1}{2}\overrightarrow{BC}=(1\,,\ 2)$

Assim, $\overrightarrow{OA}-\frac{1}{2}\overrightarrow{BC}=(1\,,\ 2)$

- $\overrightarrow{OA}=A-O$
  $\overrightarrow{OA}=(2\,,\ 0)-(0\,,\ 0)$
  $\overrightarrow{OA}=(2\,,\ 0)$
- $\overrightarrow{BC}=C-B$
  $\overrightarrow{BC}=(0\,,\ -2)-(-2\,,\ 2)$
  $\overrightarrow{BC}=(2\,,\ -4)$

Resposta: **(C)**.

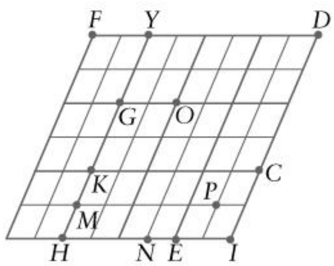
6. $\|\vec{u}\|=\sqrt{(\sqrt{3})^2+(-1)^2} \Leftrightarrow \|\vec{u}\|=\sqrt{3+1} \Leftrightarrow \|\vec{u}\|=2$

Logo, $\|\vec{u}\|^2=4$.

Resposta: **(A)**.

---

7. **Pág. 132**



8.1 **a)** Como B é o ponto médio de $[OC]$, então
$\overrightarrow{OC}=2\overrightarrow{OB}\,;\ \overrightarrow{OC}=2\vec{y}\,;$

b) $\overrightarrow{OB}+\overrightarrow{BA}=\overrightarrow{OA}\,;\ \vec{y}+\overrightarrow{BA}=\vec{x}\,;\ \overrightarrow{BA}=\vec{x}-\vec{y}\,;$

c) Temos que: $\overrightarrow{OA}+\overrightarrow{AC}=\overrightarrow{OC}\,;\ \vec{y}+\overrightarrow{AC}=2\vec{y}\,;\ \overrightarrow{AC}=2\vec{y}-\vec{x}\,;$
Por outro lado, temos que:
$\overrightarrow{OA}+\overrightarrow{AM}=\overrightarrow{OM}\,;\ \vec{x}+\frac{1}{2}\overrightarrow{AC}=\overrightarrow{OM}\,;$
$\vec{x}+\frac{1}{2}(2\vec{y}-\vec{x})=\overrightarrow{OM}$
$\overrightarrow{OM}=\frac{1}{2}\vec{x}+\vec{y}\,;$

8.2 $\overrightarrow{BH} = \frac{1}{3}\overrightarrow{BA}$; $\overrightarrow{BH} = \frac{1}{3}(\vec{x} - \vec{y})$;

8.3 $\overrightarrow{OH} = \overrightarrow{OB} + \overrightarrow{BH} = \vec{y} + \frac{1}{3}(\vec{x} - \vec{y}) = \vec{y} + \frac{1}{3}\vec{x} - \frac{1}{3}\vec{y} = \frac{1}{3}\vec{x} + \frac{2}{3}\vec{y}$

$\overrightarrow{OM} = \frac{1}{2}\vec{x} + \vec{y}$

Portanto $\overrightarrow{OH} = \frac{2}{3}\overrightarrow{OM}$, logo $\overrightarrow{OH}$ é múltiplo de $\overrightarrow{OM}$.

Como $\overrightarrow{OH}$ e $\overrightarrow{OM}$ são colineares, logo são paralelos e têm em comum o ponto O , podemos, assim, concluir que O , H e M são colineares (pertencem à mesma recta).

9.1 $\vec{u}$ e $\vec{v}$ são colineares se e só se existir um número real $k \neq 0$ tal que $\vec{u} = k\vec{v}$.

$\vec{u} = k\vec{v} \Leftrightarrow (u_1 , u_2) = k(v_1 , v_2) \Leftrightarrow$

$\Leftrightarrow (u_1 , u_2) = (kv_1 , kv_2) \Leftrightarrow$

$$\Leftrightarrow \begin{cases} kv_1 = u_1 \\ kv_2 = u_2 \end{cases} \Leftrightarrow \begin{cases} k = \dfrac{u_1}{v_1} \\ k = \dfrac{u_2}{v_2} \end{cases}$$

Como k tem de ser único temos que

$\frac{u_1}{v_1} = \frac{u_2}{v_2} \Leftrightarrow u_1 \times v_2 = u_2 \times v_1$, com $v_1 \times v_2 \neq 0$ c. q. m.

9.2 **a)** $-1 \times 1 = 5 \times \frac{1}{5} \Leftrightarrow -1 = 1$ (F)

Então os vectores $\vec{a}$ e $\vec{b}$ não são colineares;

b) $\frac{1}{3} \times 3 = \sqrt{3} \times \frac{\sqrt{3}}{3} \Leftrightarrow 1 = \frac{3}{3} \Leftrightarrow 1 = 1$ (V)

Então os vectores $\vec{c}$ e $\vec{d}$ são colineares;

c) $-\pi \times \left(-\frac{2\pi}{3}\right) = \frac{\pi}{3} \times 2\pi \Leftrightarrow \frac{2\pi^2}{3} = \frac{2\pi^2}{3}$ (V)

Então os vectores $\vec{e}$ e $\vec{f}$ são colineares.

10.1 **a)** $\overrightarrow{AB} = B - A = (-2 , 0) - (4 , 2) = (-6 , -2)$

$\overrightarrow{AC} = C - A = (0 , -3) - (4 , 2) = (-4 , -5)$

$\overrightarrow{AB} - \overrightarrow{AC} = (-6 , -2) - (-4 , -5)$

$\overrightarrow{AB} - \overrightarrow{AC} = (-2 , 3)$;

b) $\overrightarrow{AC} = (-4 , -5)$

$\overrightarrow{DB} = B - D = (-2 , 0) - (-1 , 4) = (-1 , -$

$\overrightarrow{AC} - \overrightarrow{DB} = (-4 , -5) - (-1 , -4) = (-3 , -$

$-\frac{1}{2}(\overrightarrow{AC} - \overrightarrow{DB}) = -\frac{1}{2}(-3 , -1)$

$-\frac{1}{2}(\overrightarrow{AC} - \overrightarrow{DB}) = \left(\frac{3}{2} , \frac{1}{2}\right)$;

c) $A - D = (4 , 2) - (-1 , 4) = (5 , -2)$

$D - A = (-1 , 4) - (4 , 2) = (-5 , 2)$

$3(A - D) + 2(D - A) = 3(5 , -2) + 2(-5 , 2)$

$3(A - D) + 2(D - A) = (15 , -6) + (-10 , 4)$

$3(A - D) + 2(D - A) = (5 , -2)$.

10.2 $P_{[ABCD]} = ||\overrightarrow{AD}|| + ||\overrightarrow{BD}|| + ||\overrightarrow{BC}|| + ||\overrightarrow{AC}||$

• $\overrightarrow{AD} = D - A = (-5 , 2)$

$||\overrightarrow{AD}|| = \sqrt{(-5)^2 + 2^2} = \sqrt{25 + 4} = \sqrt{29}$

• $\overrightarrow{BD} = D - B = (1 , 4)$

$||\overrightarrow{BD}|| = \sqrt{1^2 + 4^2} = \sqrt{1 + 16} = \sqrt{17}$

• $\overrightarrow{BC} = C - B = (0 , -3) - (-2 , 0) = (2 , -3)$

$||\overrightarrow{BC}|| = \sqrt{2^2 + (-3)^2} = \sqrt{4 + 9} = \sqrt{13}$

• $\overrightarrow{AC} = (-4 , -5)$

$||\overrightarrow{AC}|| = \sqrt{(-4)^2 + (-5)^2} = \sqrt{16 + 25} = \sqrt{41}$

$P_{[ABCD]} = \sqrt{29} + \sqrt{17} + \sqrt{13} + \sqrt{41} = 19{,}5$ cm (1 c. d.) .

10.3 $\overrightarrow{AB} = (-6 , -2)$; $\overrightarrow{AC} = (-4 , -5)$;

$\overrightarrow{AB} - 2\overrightarrow{AC} = (-6 , -2) - 2(-4 , -5)$

$= (-6 , -2) - (-8 , -10) = (2 , 8)$

$||\overrightarrow{AB} - 2\overrightarrow{AC}|| = \sqrt{2^2 + 8^2} \Leftrightarrow ||\overrightarrow{AB} - 2\overrightarrow{AC}|| = \sqrt{4 + 64} \Leftrightarrow$

$\Leftrightarrow ||\overrightarrow{AB} - 2\overrightarrow{AC}|| = \sqrt{68} \Leftrightarrow ||\overrightarrow{AB} - 2\overrightarrow{AC}|| = 2\sqrt{17}$.

10.4 Como o ponto E pertence ao eixo das ordenadas, então as suas coordenadas são $(0 , y)$.

$\overrightarrow{AE} = E - A = (0 , y) - (4 , 2) = (-4 , y - 2)$

Como $||\overrightarrow{AE}|| = \sqrt{17}$, vem que:

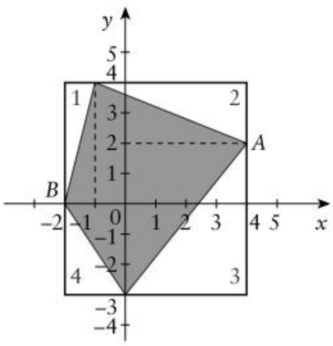
$\sqrt{(-4)^2 + (y - 2)^2} = \sqrt{17} \Rightarrow 16 + y^2 - 4y + 4 = 17 \Leftrightarrow$

$\Leftrightarrow y^2 - 4y + 20 - 17 = 0 \Leftrightarrow y^2 - 4y + 3 = 0 \Leftrightarrow$

$\Leftrightarrow y = \frac{4 \pm \sqrt{16 - 12}}{2} \Leftrightarrow y = \frac{4 \pm 2}{2} \Leftrightarrow y = 3 \vee y = 1$

Assim, $E(0 , 3)$ ou $E(0 , 1)$.

10.5



$A_{[ABCD]} = A_{\text{rectângulo}} - (A_1 + A_2 + A_3 + A_4)$

$A_1 = \frac{4 \times 1}{2} = 2$; $A_2 = \frac{5 \times 2}{2} = 5$; $A_3 = \frac{5 \times 4}{2} = 10$;

$A_4 = \frac{3 \times 2}{2} = 3$

$A_{\text{rectângulo}} = 6 \times 7 = 42$

Assim, temos que:

$A_{[ABCD]} = 42 - (2 + 5 + 10 + 3) \Leftrightarrow A_{[ABCD]} = 22$ cm^2 .