1.1 $1 - x = \frac{1}{2} \Leftrightarrow -x = -\frac{1}{2} \Leftrightarrow x = \frac{1}{2}$ **Pág. 43**

$S = \left\{\frac{1}{2}\right\}$, possível e determinada.

1.2 $-\frac{1}{2}x = 0 \Leftrightarrow x = 0$

$S = \{0\}$, possível e determinada.

1.3 $-x - (-x + 2) = \frac{1}{4} \Leftrightarrow -x + x - 2 = \frac{1}{4} \Leftrightarrow 0x = \frac{9}{4}$

$S = \{\ \}$, impossível.

1.4 $-x + 3 - 7 = x \Leftrightarrow -2x = 4 \Leftrightarrow x = -2$

$S = \{-2\}$, possível e determinada.

1.5 $x - 2 = -(2 + x) \Leftrightarrow x - 2 = -2 - x \Leftrightarrow 2x = 0 \Leftrightarrow x = 0$

$S = \{0\}$, possível e determinada.

2.1 $-\frac{1}{3}x = 0 \Leftrightarrow x = 0$ **Pág. 44**

$S = \{0\}$

2.2 $5x^2 - 4 = 0 \Leftrightarrow x^2 = \frac{4}{5} \Leftrightarrow x = \sqrt{\frac{4}{5}} \vee x = -\sqrt{\frac{4}{5}}$

$\Leftrightarrow x = \frac{2}{\sqrt{5}} \vee x = -\frac{2}{\sqrt{5}}$

$S = \left\{-\frac{2}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right\}$

2.3 $6x^2 + 4 = 0 \Leftrightarrow x^2 = -\frac{4}{6} \Leftrightarrow x = \sqrt{-\frac{4}{6}}$

$x = \sqrt{-\frac{4}{6}}$ não é um número real.

Uma vez que $\forall x \in \mathbb{R},\ x^2 \geq 0$, a equação é impossível.

$S = \{\ \}$

2.4 $-x^2 = 7x \Leftrightarrow x^2 + 7x = 0 \Leftrightarrow x(x + 7) = 0$

$\Leftrightarrow x = 0 \vee x + 7 = 0 \Leftrightarrow x = 0 \vee x = -7$

$S = \{-7, 0\}$

2.5 $x^2 - 7x + 10 = 0 \Leftrightarrow x = \frac{7 \pm \sqrt{49 - 40}}{2} \Leftrightarrow x = \frac{7 \pm \sqrt{9}}{2}$

$\Leftrightarrow x = \frac{7 \pm 3}{2} \Leftrightarrow x = 5 \vee x = 2$

$S = \{2, 5\}$

3.1 $\frac{2}{3}x^3 = 0 \Leftrightarrow x^3 = 0 \Leftrightarrow x = 0$

$S = \{0\}$

3.2 $3x^3 + 1 = 0 \Leftrightarrow x^3 = -\frac{1}{3} \Leftrightarrow x = \sqrt[3]{-\frac{1}{3}}$

$S = \left\{\sqrt[3]{-\frac{1}{3}}\right\}$

3.3 $2x^3 - 14x^2 + 24x = 0 \Leftrightarrow 2x\left(x^2 - 7x + 12\right) = 0$

$\Leftrightarrow 2x = 0 \vee x^2 - 7x + 12 = 0$

$\Leftrightarrow x = 0 \vee (x - 6)(x - 1)$

$\Leftrightarrow x = 0 \vee x = 6 \vee x = 1$

$S = \{0, 1, 6\}$

Cálculo auxiliar

$x^2 - 7x + 12 = 0$

$\Leftrightarrow x = \frac{7 \pm \sqrt{49 - 24}}{2} \Leftrightarrow$

$x = \frac{7 \pm 5}{2} \Leftrightarrow x = 5 \vee x = 2$

4.1 $x^4 - 6x^2 + 8 = 0 \Leftrightarrow$ **Pág. 45**

Para $x^2 = t$, a equação dada transforma-se em:

$t^2 - 6t + 8 = 0 \Leftrightarrow t = \frac{6 \pm \sqrt{36 - 32}}{2}$

$\Leftrightarrow t = \frac{6 \pm 2}{2} \Leftrightarrow t = 4 \vee t = 2$

Tem-se que:

$x^2 = 4 \vee x^2 = 2 \Leftrightarrow x = -2 \vee x = 2 \vee x = \sqrt{2} \vee x = -\sqrt{2}$

$S = \left\{-2, -\sqrt{2}, \sqrt{2}, 2\right\}$

4.2 $x^4 - 2x^2 - 3 = 0 \Leftrightarrow$

Para $x^2 = t$, a equação dada transforma-se em:

$t^2 - 2t - 3 = 0 \Leftrightarrow t = \frac{2 \pm \sqrt{4 + 12}}{2}$

$\Leftrightarrow t = \frac{2 \pm 4}{2} \Leftrightarrow t = 3 \vee t = -1$

Tem-se que:

$x^2 = 3 \vee x^2 = -1 \Leftrightarrow x = -\sqrt{3} \vee x = \sqrt{3}$

Note-se que $x^2 = -1$ é impossível, já que $\forall x, x^2 \geq 0$

$S = \left\{-\sqrt{3}, \sqrt{3}\right\}$

$S = \{-3, 3\}$

5.1 $-x > \frac{1}{2} + 2x \Leftrightarrow -3x > \frac{1}{2} \Leftrightarrow 3x < -\frac{1}{2} \Leftrightarrow x < -\frac{1}{6}$

$S = \left]-\infty, \frac{1}{6}\right[$

5.2 $1 - \frac{x - 1}{2} > \frac{x + 3}{5} \Leftrightarrow \frac{2 - x + 1}{2} > \frac{x + 3}{5} \Leftrightarrow \frac{3 - x}{2} > \frac{x + 3}{5}$

$\Leftrightarrow 5(3 - x) > 2(x + 3) \Leftrightarrow 15 - 5x > 2x + 6 \Leftrightarrow$

$\Leftrightarrow -7x > -9 \Leftrightarrow 7x < 9 \Leftrightarrow x < \frac{9}{7}$

$S = \left]-\infty, \frac{9}{7}\right[$

6.1 $(x - 2)(x - 1) > 0$ **Pág. 46**

- zero de $x - 2$ é 2
- zero de $x - 1$ é 1

x	**- ∞**	**1**		**2**	**+ ∞**
(x-2)	-	-	-	0	+
(x-1)	-	0	+	+	+
(x-1) (x+1)	+	0	-	0	+

$S = \left]-\infty, 1\right[\cup \left]2, +\infty\right[$

6.2 $x(x - 3) < 0$

- zero de x é 0
- zero de $x - 3$ é 3

x	**- ∞**	**0**		**3**	**+ ∞**
x	-	0	+	+	+
(x-3)	-	-	-	0	+
(x-1) (x+1)	+	0	-	0	+

$S = \left]0, 3\right[$

6.3 $x^2-6x\geq 0 \Leftrightarrow x(x-6)\geq 0$ **Pág. 43**

- zero de x é 0
- zero de $x-6$ é 6

x	$-\infty$	0		6	$+\infty$
X	-	0	+	+	+
$(x$-6)	-	-	-	0	+
(x-1) (x+1)	+	0	-	0	+

$S=\left]-\infty,0\right]\cup\left[6,+\infty\right[$

7.1 $\sqrt{2}\,x+3$ é uma expressão racional porque é definida por um polinómio; $\sqrt{2x}+3$ não é uma expressão racional porque a variável x figura no radicando. **Pág. 47**

7.2 a) $2x-2=2(x-1)$

b) $x^2-4=(x-2)(x+2)$

c) $1-x^2=(1-x)(1+x)$

d) $\dfrac{x^2}{4}-49=\left(\dfrac{x}{2}-7\right)\left(\dfrac{x}{2}+7\right)$

e) $x^2-4x+3 \Leftrightarrow x=\dfrac{4\pm\sqrt{16-12}}{2} \Leftrightarrow x=\dfrac{4\pm\sqrt{4}}{2}$

$\Leftrightarrow x=\dfrac{4\pm 2}{2} \Leftrightarrow x=3 \vee x=1$

Então, $x^2-4x+3=(x-1)(x-3)$

f) $2x^2-11x+5$

$\Leftrightarrow x=\dfrac{11\pm\sqrt{121-40}}{4} \Leftrightarrow x=\dfrac{11\pm\sqrt{81}}{4}$

$\Leftrightarrow x=\dfrac{11\pm 9}{2} \Leftrightarrow x=10 \vee x=1$

Então, $2x^2-11x+5=(x-5)(2x-1)$

g) $x^3-x^2=x^2(x-1)$

h) $x^3-x=x\left(x^2-1\right)=x(x-1)(x+1)$

7.3 a) $\dfrac{1}{2x-2}$

$D=\{x\in\mathbb{R}: 2x-2\neq 0\}$

$2x-2=0 \Leftrightarrow x=1$

$D=\mathbb{R}\setminus\{1\}$

b) $\dfrac{3x}{x^2-4}$

$D=\{x\in\mathbb{R}: x^2-4\neq 0\}$

$x^2-4=0 \Leftrightarrow (x-2)(x+2)=0 \Leftrightarrow x=2 \vee x=-2$

$D=\mathbb{R}\setminus\{-2,2\}$

c) $\dfrac{x^3}{1-x^2}$

$D=\{x\in\mathbb{R}: 1-x^2\neq 0\}$

$1-x^2=0 \Leftrightarrow (1-x)(1+x)=0 \Leftrightarrow x=1 \vee x=-1$

$D=\mathbb{R}\setminus\{-1,1\}$

d) $\dfrac{3x^5-1}{\dfrac{x^2}{4}-49}$

$D=\left\{x\in\mathbb{R}: \dfrac{x^2}{4}-49\neq 0\right\}$

$\dfrac{x^2}{4}-49=0 \Leftrightarrow \left(\dfrac{x}{2}-7\right)\left(\dfrac{x}{2}+7\right)=0 \Leftrightarrow x=14 \vee x=-14$

$D=\mathbb{R}\setminus\{-14,14\}$

e) $\dfrac{1}{x^2-4x+3}$

$D=\{x\in\mathbb{R}: x^2-4x+3\neq 0\}$

$x^2-4x+3=0 \Leftrightarrow x=\dfrac{4\pm\sqrt{16-12}}{2} \Leftrightarrow x=\dfrac{4\pm 2}{2}$

$\Leftrightarrow x=3 \vee x=1$

$D=\mathbb{R}\setminus\{1,3\}$

f) $\dfrac{1}{2x^2-11x+5}$

$D=\{x\in\mathbb{R}: 2x^2-11x+5\neq 0\}$

$2x^2-11x+5=0 \Leftrightarrow x=\dfrac{11\pm\sqrt{121-40}}{4} \Leftrightarrow x=\dfrac{11\pm 9}{4}$

$\Leftrightarrow x=5 \vee x=\dfrac{1}{2}$

$D=\mathbb{R}\setminus\left\{\dfrac{1}{2},5\right\}$

g) $\dfrac{3x+1}{x^3-x^2}$

$D=\{x\in\mathbb{R}: x^3-x^2\neq 0\}$

$x^3-x^2=0 \Leftrightarrow x^2(x-1)=0 \Leftrightarrow x^2=0 \vee x-1=0$

$\Leftrightarrow x=0 \vee x=1$

$D=\mathbb{R}\setminus\{0,1\}$

h) $\dfrac{3x^2-7}{x^3-x}$

$D=\{x\in\mathbb{R}: x^3-x\neq 0\}$

$x^3-x=0 \Leftrightarrow x\left(x^2-1\right)=0 \Leftrightarrow x(x-1)(x+1)=0$

$\Leftrightarrow x=0 \vee x-1=0 \vee x+1=0 \Leftrightarrow x=0 \vee x=1 \vee x=-1$

$D=\mathbb{R}\setminus\{-1,0,1\}$

i) $\dfrac{3x^2+1}{4x^2-4x+6}$

$D=\{x\in\mathbb{R}: 4x^2-4x+6\neq 0\}$

$4x^2-4x+6=0 \Leftrightarrow x=\dfrac{4\pm\sqrt{16-96}}{8} \Leftrightarrow x=\dfrac{4\pm\sqrt{-80}}{8}$

Uma vez que x^2-4x+6 não tem raízes reais, então é, $\forall x$, diferente de zero. Logo,

$D=\mathbb{R}$

j) $\dfrac{2x}{3x^2-6x-9}$

$D=\{x\in\mathbb{R}: 3x^2-6x-9\neq 0\}$

$3x^2-6x-9=0 \Leftrightarrow x=\dfrac{6\pm\sqrt{36+108}}{6} \Leftrightarrow x=\dfrac{6\pm 12}{6}$

$\Leftrightarrow x=3 \vee x=-1$

$D=\mathbb{R}\setminus\{-1,3\}$

k) $\frac{3x^6+1}{x^3-4x^2-5x}$

$D=\left\{x\in\mathbb{R}: x^3-4x^2-5x\neq 0\right\}$

$x^3-4x^2-5x=0 \Leftrightarrow x\left(x^2-4x-5\right)=0$

$\Leftrightarrow x=0 \vee x^2-4x-5=0$

$\Leftrightarrow x=\frac{4\pm\sqrt{16+20}}{2} \Leftrightarrow x=\frac{4\pm 6}{2} \Leftrightarrow x=5 \vee x=-1$

Então, $x^3-4x^2-5x=0 \Leftrightarrow x=0 \vee x=-1 \vee x=5$

$D=\mathbb{R}\setminus\{-1,0,5\}$

l) $\frac{2x}{x^4-2x^2-3}$

$D=\left\{x\in\mathbb{R}: x^4-2x^2-3\neq 0\right\}$

$x^4-2x^2-3=0$

Fazendo $x^2=t$, esta equação transforma-se em:

$t^2-2t-3=0 \Leftrightarrow t=\frac{2\pm\sqrt{4+12}}{2} \Leftrightarrow$

$\Leftrightarrow t=\frac{2\pm 4}{2} \Leftrightarrow t=3 \vee t=-1$

Tem-se que:

$x^2=3 \vee x^2=-1 \Leftrightarrow x=-\sqrt{3} \vee x=\sqrt{3}$, uma vez que $x^2\geq 0$

$D=\mathbb{R}\setminus\left\{-\sqrt{3},\sqrt{3}\right\}$

8.1 $\frac{x-3}{9-x^2}=\frac{x-3}{(3-x)(3+x)}=\frac{-(3-x)}{(3-x)(3+x)}=\frac{-1}{x+3}$ **Pág. 48**

$D=\left\{x\in\mathbb{R}: 9-x^2\neq 0\right\}$

$9-x^2=0 \Leftrightarrow x^2=9 \Leftrightarrow x=\pm\sqrt{9} \Leftrightarrow x=-3 \vee x=3$

$D=\mathbb{R}\setminus\{-3,3\}$

8.2 $\frac{x-5}{x^2+25-10x}=$

$=\frac{x-5}{(x-5)^2}=\frac{1}{x-5}$

Cálculo auxiliar:

$x=\frac{10\pm\sqrt{100-100}}{2} \Leftrightarrow$

$\Leftrightarrow x=5$

$D=\left\{x\in\mathbb{R}: x^2+25-10x\neq 0\right\}$

$x^2+25-10x=0 \Leftrightarrow (x-5)^2=0 \Leftrightarrow x=5$

$D=\mathbb{R}\setminus\{5\}$

8.3 $\frac{\frac{x+2}{2}}{\frac{1}{x}}=\frac{x(x+2)}{2}$

$D=\left\{x\in\mathbb{R}: x\neq 0\right\}$

$D=\mathbb{R}\setminus\{0\}$

8.4 $\frac{3x+x^2}{x+3}=\frac{x(3+x)}{x+3}=x$

$D=\left\{x\in\mathbb{R}: x+3\neq 0\right\}$

$x+3=0 \Leftrightarrow x=-3$

$D=\mathbb{R}\setminus\{-3\}$

8.5 $\frac{(x-5)^2}{25-x^2}=\frac{(x-5)^2}{(5-x)(5+x)}=\frac{(x-5)^2}{-(x-5)(5+x)}=\frac{5-x}{5+x}$

$D=\left\{x\in\mathbb{R}: 25-x^2\neq 0\right\}$

$25-x^2=0 \Leftrightarrow x=\pm\sqrt{25} \Leftrightarrow x=-5 \vee x=5$

$D=\mathbb{R}\setminus\{-5,5\}$

8.6 $\frac{1}{1-x}-\frac{2}{x}=\frac{x-2(1-x)}{x(1-x)}=\frac{x-2+2x}{x(1-x)}=\frac{3x-2}{x(1-x)}$

$D=\left\{x\in\mathbb{R}: 1-x\neq 0 \wedge x\neq 0\right\}$

$1-x=0 \vee x=0 \Leftrightarrow x=1 \vee x=0$

$D=\mathbb{R}\setminus\{0,1\}$

8.7 $\frac{x^2-3x-4}{x^2-4x}=\frac{(x-4)(x+1)}{x(x-4)}=$

$=\frac{x+1}{x}$

Cálculo auxiliar:

$x=\frac{3\pm\sqrt{9+16}}{2} \Leftrightarrow$

$\Leftrightarrow x=4 \vee x=-1$

$D=\left\{x\in\mathbb{R}: x^2-4x\neq 0\right\}$

$x^2-4x=0 \Leftrightarrow x(x-4)=0 \Leftrightarrow$

$\Leftrightarrow x=0 \vee x-4=0 \Leftrightarrow x=0 \vee x=4$

$D=\mathbb{R}\setminus\{0,4\}$

8.8 $\frac{x^5-x^2}{x^4+x^3+x^2}=\frac{x^2\left(x^3-1\right)}{x^2\left(x^2+x+1\right)}=\frac{x^3-1}{x^2+x+1}=$

$=\frac{(x-1)\left(x^2+x+1\right)}{x^2+x+1}=x-1$

$D=\left\{x\in\mathbb{R}: x^4+x^3+x^2\neq 0\right\}$

$x^4+x^3+x^2=0 \Leftrightarrow x^2\left(x^2+x+1\right)=0 \Leftrightarrow$

$\Leftrightarrow x^2=0 \vee x^2+x+1=0 \Leftrightarrow$

$\Leftrightarrow x=0 \vee x=\frac{-1\pm\sqrt{1-4}}{2}=\frac{-1\pm\sqrt{-3}}{2}$

$D=\mathbb{R}\setminus\{0\}$

8.9 $\frac{x+\frac{1}{2}x}{x-\frac{1}{x}}=\frac{\frac{3}{2}x}{\frac{x^2-1}{x}}=\frac{\frac{3}{2}x^2}{x^2-1}=\frac{3x^2}{2\left(x^2-1\right)}$

$D=\left\{x\in\mathbb{R}: x-\frac{1}{x}\neq 0 \wedge x\neq 0\right\}$

$x-\frac{1}{x}=0 \vee x=0 \Leftrightarrow \frac{x^2-1}{x}=0 \vee x=0 \Leftrightarrow$

$\Leftrightarrow x^2-1=0 \vee x=0 \Leftrightarrow x=-1 \vee x=-1 \vee x=0$

$D=\mathbb{R}\setminus\{-1,0,1\}$

9.1 $\frac{x}{4}\times\frac{-4}{5x^3}=\frac{-\cancel{4}x}{\cancel{4}\times 5x^3}=\frac{-x}{5x^3}=-\frac{1}{5x^2}$ **Pág. 49**

$D=\left\{x\in\mathbb{R}: 5x^3\neq 0\right\}$

$D=\mathbb{R}\setminus\{0\}$

9.2 $\frac{x^2+3x}{2-x}\times\frac{x^2-4}{x^2-9}=\frac{x(x+3)\left(x^2-4\right)}{(2-x)(x-3)(x+3)}=$

$=\frac{x(x-2)(x+2)}{(2-x)(x-3)}=\frac{x(2-x)(x+2)}{-(2-x)(x-3)}=\frac{x^2+2x}{3-x}$

$D=\left\{x\in\mathbb{R}: 2-x\neq 0 \wedge x^2-9\neq 0\right\}$

$2-x=0 \vee x^2-9=0 \Leftrightarrow x=2 \vee x=-3 \vee x=3$

$D=\mathbb{R}\setminus\{-3,2,3\}$

9.3 $\frac{2x}{x^2+2x+1}\times\frac{1-x^2}{x^2}=$

$=\frac{2x\left(1-x^2\right)}{\left(x^2+2x+1\right)x^2}=$

Cálculo auxiliar:

$x=\frac{-2\pm\sqrt{4-4}}{2} \Leftrightarrow$

$\Leftrightarrow x=-1$

$=\frac{2(1-x)(1+x)}{(x+1)^2 x}=\frac{2(1-x)}{(x+1)x}=\frac{2-2x}{x^2+x}$

$D=\left\{x\in\mathbb{R}: x^2+2x+1\neq 0 \wedge x^2\neq 0\right\}$

$x^2+2x+1=0 \vee x^2=0 \Leftrightarrow x=-1 \vee x=0$

$D=\mathbb{R}\setminus\{-1,0\}$

9.4 $\frac{x^2+4x+1}{x-2}\times\frac{x^2-4x+1}{x+2}=\frac{(x+2)^2(x-2)^2}{(x-2)(x+2)}=$

$=(x-2)(x+2)=x^2+2x-2x-4=x^2-4$

$D=\{x\in\mathbb{R}: x-2\neq 0 \wedge x+2\neq 0\}$

$x-2=0 \vee x+2=0 \Leftrightarrow x=2 \vee x=-2$

$D=\mathbb{R}\setminus\{-2,2\}$

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10.1 $(-3x):\dfrac{2}{x+1}=(-3x)\times\dfrac{x+1}{2}=-\dfrac{3x^2+3x}{2}$

$D=\{x\in\mathbb{R}: x+1\neq 0\}$

$x+1=0 \Leftrightarrow x=-1$

$D=\mathbb{R}\setminus\{-1\}$

10.2 $\dfrac{x^4-1}{x^4}:\dfrac{x^2+1}{3x}=\dfrac{x^4-1}{x^4}\times\dfrac{3x}{x^2+1}=$

$=\dfrac{3x(x^4-1)}{x^4}\times\dfrac{1}{x^2+1}=\dfrac{3(x^2-1)(x^2+1)}{x^3(x^2+1)}=$

$=\dfrac{3(x^2-1)}{x^3}=\dfrac{3x^2-3}{x^3}$

$D=\left\{x\in\mathbb{R}: x^4\neq 0 \wedge \dfrac{x^2+1}{3x}\neq 0\right\}$

$D=\mathbb{R}\setminus\{0\}$

10.3 $\dfrac{x^2-25}{15x}:\dfrac{x^2+10x+25}{9x^2}=$

$=\dfrac{x^2-25}{15x}\times\dfrac{9x^2}{x^2+10x+25}=$

$=\dfrac{9x^2(x-5)(x+5)}{15x(x+5)^2}=$

$=\dfrac{9x\cdot(x-5)}{15(x+5)}=\dfrac{3x^2-15x}{5x+25}$

$D=\left\{x\in\mathbb{R}: 15x\neq 0 \wedge \dfrac{x^2+10x+25}{9x^2}\neq 0\right\}$

$D=\mathbb{R}\setminus\{-5,0\}$

Cálculo auxiliar:

$x^2+10x+25=0 \Leftrightarrow$

$\Leftrightarrow x=\dfrac{-10\pm\sqrt{100-100}}{2} \Leftrightarrow$

$\Leftrightarrow x=-5$

10.4 $\dfrac{x^2+4x+3}{x^2-5x+4}:\dfrac{x+3}{x-4}=$

$=\dfrac{x^2+4x+3}{x^2-5x+4}\times\dfrac{x-4}{x+3}=$

$=\dfrac{(x+1)(x+3)(x-4)}{(x^2-5x+4)(x+3)}=$

$=\dfrac{(x+1)(x-4)}{(x-4)(x-1)}=$

$=\dfrac{(x+1)}{(x-1)}$

$D=\left\{x\in\mathbb{R}: x^2-5x+4\neq 0 \wedge \dfrac{x+3}{x-4}\neq 0\right\}$

$D=\mathbb{R}\setminus\{-3,1,4\}$

Cálculo auxiliar:

$x^2+4x+3=0 \Leftrightarrow$

$\Leftrightarrow x=\dfrac{-4\pm\sqrt{16-12}}{2} \Leftrightarrow$

$\Leftrightarrow x=-1 \vee x=-3$

$x^2-5x+4=0 \Leftrightarrow$

$\Leftrightarrow x=\dfrac{5\pm\sqrt{25-16}}{2} \Leftrightarrow$

$\Leftrightarrow x=4 \vee x=1$

11.1 $\dfrac{x}{x-2}+\dfrac{2x+1}{x+2}-\dfrac{2x^2}{x^2-4}=$

$=\dfrac{x(x+2)}{x^2-4}+\dfrac{(2x+1)(x-2)}{x^2-4}-\dfrac{2x^2}{x^2-4}=$

$=\dfrac{x^2+2x+2x^2-4x+x-2-2x^2}{x^2-4}=$

$=\dfrac{x^2-x-2}{x^2-4}=\dfrac{(x-2)(x+1)}{(x-2)(x+2)}=\dfrac{(x+1)}{(x+2)}$

$D=\{x\in\mathbb{R}: x-2\neq 0 \vee x+2\neq 0 \vee x^2-4\neq 0\}$

$D=\mathbb{R}\setminus\{-2,2\}$

11.2 $\dfrac{x^2-1}{x}-\dfrac{x^2}{x+1}+\dfrac{1}{x^2+x}=$

$=\dfrac{(x^2-1)(x+1)}{x^2+x}-\dfrac{x^2\cdot x}{x^2+x}+\dfrac{1}{x^2+x}=$

$=\dfrac{x^3+x^2-x-1-x^3+1}{x^2+x}=\dfrac{x^2-x}{x^2+x}=$

$=\dfrac{x(x-1)}{x(x+1)}=\dfrac{(x-1)}{(x+1)}$

$D=\{x\in\mathbb{R}: x\neq 0 \wedge x+1\neq 0 \wedge x^2+x\neq 0\}$

$D=\mathbb{R}\setminus\{-1,0\}$

11.3 $\dfrac{2x}{x-2}-\dfrac{2x+3}{2x^2-x-6}=$

$=\dfrac{2x}{x-2}-\dfrac{2x+3}{2(x-2)\left(x+\frac{3}{2}\right)}=$

$=\dfrac{4x\cdot(2x+3)}{x-2}-\dfrac{2x+3}{2(x-2)(2x+3)}=$

$=\dfrac{4x\cdot(2x+3)-(2x+3)}{2(x-2)(2x+3)}=\dfrac{2(2x+3)(2x-1)}{2(x-2)(2x+3)}=$

$=\dfrac{2x-1}{x-2}$

$D=\{x\in\mathbb{R}: x-2\neq 0 \wedge 2x^2-x-6\neq 0\}$

$D=\mathbb{R}\setminus\left\{-\dfrac{3}{2},2\right\}$

Cálculo auxiliar:

$2x^2-x-6=0 \Leftrightarrow$

$\Leftrightarrow x=\dfrac{1\pm\sqrt{1+48}}{4} \Leftrightarrow$

$\Leftrightarrow x=2 \vee x=-\dfrac{3}{2}$

11.4 $\dfrac{4}{x^2-4}-\dfrac{2x}{2-x}+\dfrac{3}{x+2}=\dfrac{4}{x^2-4}+\dfrac{2x}{x-2}+\dfrac{3}{x+2}=$

$=\dfrac{4}{x^2-4}+\dfrac{2x\cdot(x+2)}{x^2-4}+\dfrac{3(x-2)}{x^2-4}=$

$=\dfrac{4+2x^2+4x+3x-6}{x^2-4}=\dfrac{2x^2+7x-2}{x^2-4}$

$D=\{x\in\mathbb{R}: x^2-4\neq 0 \wedge 2-x\neq 0 \wedge x+2\neq 0\}$

$D=\mathbb{R}\setminus\{-2,2\}$

11.5 $\dfrac{3x-1}{(x-1)^2}+\dfrac{1}{1-x^2}=\dfrac{3x-1}{(x-1)(x-1)}-\dfrac{1}{x^2-1}=$

$=\dfrac{3x-1}{(x-1)(x-1)}-\dfrac{1}{(x-1)(x+1)}=$

$=\dfrac{(3x-1)(x+1)}{(x-1)(x-1)(x+1)}-\dfrac{x-1}{(x-1)(x-1)(x+1)}=$

$=\dfrac{3x^2+3x-x-1-x+1}{(x-1)(x-1)(x+1)}=\dfrac{3x^2+x}{(x-1)^2(x+1)}$

$D=\{x\in\mathbb{R}: (x-1)^2\neq 0 \wedge 1-x^2\neq 0\}$

$D=\mathbb{R}\setminus\{-1,1\}$

11.6 $\dfrac{4}{4x^2-9}-\dfrac{3x}{4x^2+12x+9}+\dfrac{1}{2x-3}=$

$=\dfrac{4}{(2x-3)(2x+3)}-\dfrac{3x}{4\left(x+\frac{3}{2}\right)\left(x+\frac{3}{2}\right)}+\dfrac{1}{2x-3}=$

$=\dfrac{4}{(2x-3)(2x+3)}-\dfrac{3x}{4(2x+3)(2x+3)}+\dfrac{1}{2x-3}=$

$=\dfrac{4(2x+3)-3x\cdot(2x-3)+(2x+3)(2x+3)}{(2x-3)(2x+3)^2}=$

$=\dfrac{8x+12-6x^2+9x+4x^2+6x+6x+9}{(2x-3)(2x+3)^2}=$

$=\dfrac{-2x^2+29x+21}{(2x+3)^2(2x-3)}$

$D=\{x\in\mathbb{R}: 4x^2-9\neq 0 \wedge 4x^2+12x+9\neq 0 \wedge 2x-3\neq 0\}$

$D=\mathbb{R}\setminus\left\{-\dfrac{3}{2},\dfrac{3}{2}\right\}$

Cálculo auxiliar:

$4x^2+12x+9=0 \Leftrightarrow$

$\Leftrightarrow x=\dfrac{-12\pm\sqrt{144-144}}{8} \Leftrightarrow x=-\dfrac{3}{2}$

11.7 $\dfrac{3}{x^3-9x}+\dfrac{2}{3(x-3)^2}+\dfrac{1}{x^2+3x}=$

$=\dfrac{3}{x\cdot\left(x^2-9\right)}+\dfrac{2}{3(x-3)(x-3)}+\dfrac{1}{x\cdot(x+3)}=$

$=\dfrac{3}{x\cdot(x-3)(x+3)}+\dfrac{2}{3(x-3)(x-3)}+\dfrac{1}{x\cdot(x+3)}=$

$=\dfrac{3\times3(x-3)}{x\cdot(x-3)(x+3)}+\dfrac{2x.(x+3)}{3(x-3)(x-3)}+\dfrac{3(x-3)(x-3)}{x\cdot(x+3)}=$

$=\dfrac{9x-27+2x^2+6x+3x^2-9x-9x+27}{3x\cdot(x-3)^2(x+3)}=$

$=\dfrac{2x^2+6x+3x^2-9x}{3x\cdot(x-3)^2(x+3)}=\dfrac{5x^2-3x}{3x\cdot(x-3)^2(x+3)}$

$=\dfrac{x(5x-3)}{3x\cdot(x-3)^2(x+3)}=\dfrac{(5x-3)}{3(x-3)^2(x+3)}$

$D=\left\{x\in\mathbb{R}:x^3-9x\neq0\wedge3(x-3)^2\neq0\wedge x^2+3x\neq0\right\}$

$\bullet x^3-9x=0\Leftrightarrow x\left(x^2-9\right)=0\Leftrightarrow x=0\vee x=-3\vee x=3$

$\bullet 3(x-3)^2=0\Leftrightarrow(x-3)(x+3)=0\Leftrightarrow x=3\vee x=-3$

$\bullet x^2+3x=0\Leftrightarrow x\cdot(x+3)\Leftrightarrow x=0\vee x=-3$

$D=\mathbb{R}\setminus\{-3,0,3\}$

11.8 $\dfrac{2}{x}\times\dfrac{x^2}{6}=\dfrac{2x^2}{6x}=\dfrac{x}{3}$

$D=\{x\in\mathbb{R}:x\neq0\}$

$D=\mathbb{R}\setminus\{0\}$

11.9 $\dfrac{x^2-16}{x^4}\times\dfrac{x^2}{x^2+4x}=\dfrac{(x-4)(x+4)}{x^2\cdot\left(x^2+4x\right)}=$

$=\dfrac{(x-4)(x+4)}{x^2\cdot x\cdot(x+4)}=\dfrac{(x-4)}{x^3}$

$D=\left\{x\in\mathbb{R}:x^4\neq0\wedge x^2+4x\neq0\right\}$

$\bullet x^4=0\Leftrightarrow x=0$

$\bullet x^2+4x=0\Leftrightarrow x.(x+4)=0\Leftrightarrow x=0\vee x=-4$

$D=\mathbb{R}\setminus\{-4,0\}$

11.10 $\dfrac{(x+1)\times5}{x^2-1}=\dfrac{(x+1)\times5}{(x-1)(x+1)}=\dfrac{5}{(x-1)}$

$D=\left\{x\in\mathbb{R}:x^2-1\neq0\right\}$

$D=\mathbb{R}\setminus\{-1,1\}$

11.11 $(x+1):\dfrac{5x^2-5}{x^2}=(x+1)\times\dfrac{x^2}{5x^2-5}$

$=\dfrac{(x+1)\cdot x^2}{5\left(x^2-1\right)}=\dfrac{(x+1)\cdot x^2}{5(x-1)(x+1)}=\dfrac{x^2}{(x-1)}$

$D=\left\{x\in\mathbb{R}:\dfrac{5x^2-5}{x^2}\neq0\right\}$

$\bullet x^2=0\Leftrightarrow x=0$

$\bullet 5x^2-5=0\Leftrightarrow5\left(x^2-1\right)\Leftrightarrow x=-1\vee x=1$

$D=\mathbb{R}\setminus\{-1,0,1\}$

11.12 $\dfrac{x^2-9}{2x}:\dfrac{x^2+6x+9}{4x^2}=$

$=\dfrac{x^2-9}{2x}\times\dfrac{4x^2}{x^2+6x+9}=$

$=\dfrac{2\left(x^2-9\right)\cdot x}{x^2+6x+9}=$

$=\dfrac{2(x-3)(x+3)\cdot x}{(x+3)^2}=\dfrac{2x\cdot(x-3)}{x+3}$

$D=\left\{x\in\mathbb{R}:2x\neq0\wedge\dfrac{x^2+6x+9}{4x^2}\neq0\right\}$

$D=\mathbb{R}\setminus\{-3,0\}$

Cálculo auxiliar:

$x^2+6x+9=0\Leftrightarrow$

$\Leftrightarrow x=\dfrac{-6\pm\sqrt{36-36}}{2}\Leftrightarrow$

$\Leftrightarrow x=-3$

11.13 $\dfrac{2x^2-3x+1}{2x^2-x-10}:\dfrac{x-1}{x+2}=$

$=\dfrac{2x^2-3x+1}{2x^2-x-10}\times\dfrac{x+2}{x-1}=$

$=\dfrac{2\left(x-\frac{1}{2}\right)(x-1)}{2\left(x-\frac{5}{2}\right)(x+2)}\times\dfrac{x+2}{x-1}=$

$=\dfrac{(2x-1)(x-1)(x+2)}{\left(2x-\frac{5}{2}\right)(x-1)(x+2)}=$

$=\dfrac{2x-1}{2x-5}$

$D=\left\{x\in\mathbb{R}:2x^2-x-10\neq0\wedge\dfrac{x-1}{x+2}\neq0\right\}$

$D=\mathbb{R}\setminus\left\{-2,\dfrac{5}{2},1\right\}$

Cálculo auxiliar:

$2x^2-3x+1=0\Leftrightarrow$

$\Leftrightarrow x=\dfrac{3\pm\sqrt{9-8}}{4}\Leftrightarrow$

$\Leftrightarrow x=1\vee x=\dfrac{1}{2}$

$2x^2-x-10=0\Leftrightarrow$

$\Leftrightarrow x=\dfrac{1\pm\sqrt{1+80}}{4}\Leftrightarrow$

$\Leftrightarrow x=\dfrac{5}{2}\vee x=-2$

12.1 $x-\dfrac{1}{x}=0\Leftrightarrow\dfrac{x^2-1}{x}=0$ **Pág. 53**

$\Leftrightarrow x^2-1=0\wedge x\neq0\Leftrightarrow x=-1\vee x=1$

$S=\{-1,1\}$

12.2 $\dfrac{x^2-4}{x-2}=0\Leftrightarrow\dfrac{(x-2)(x+2)}{(x-2)}=0$

$\Leftrightarrow(x-2)(x+2)=0\wedge x-2\neq0$

$\Leftrightarrow x-2=0\vee x+2=0\wedge x\neq2\Leftrightarrow x=-2$

$S=\{-2\}$

12.3 $x+\dfrac{2}{2x-6}+\dfrac{1}{x-3}=2$

$\Leftrightarrow x+\dfrac{2}{2(x-3)}+\dfrac{1}{x-3}-2=0$

$\Leftrightarrow x+\dfrac{1}{(x-3)}+\dfrac{1}{x-3}-2=0$

$\Leftrightarrow\dfrac{x(x-3)+1+1-2(x-3)}{(x-3)}=0$

$\Leftrightarrow\dfrac{x(x-3)+1+1-2(x-3)}{(x-3)}=0$

$\Leftrightarrow\dfrac{x^2-3x+2-2x+6}{(x-3)}=0\Leftrightarrow\dfrac{x^2-5x+8}{(x-3)}=0$

$\Leftrightarrow x^2-5x+8=0\wedge x-3\neq0$

$\Leftrightarrow x=\dfrac{5\pm\sqrt{25-32}}{2}\Leftrightarrow x=\dfrac{5\pm\sqrt{-7}}{2}$

$S=\{\ \}$

12.4 $1-\dfrac{x}{4x^2-1}=2+\dfrac{x}{1-4x^2}\Leftrightarrow$

$\Leftrightarrow-1-\dfrac{x}{4x^2-1}=-\dfrac{x}{4x^2-1}\Leftrightarrow-1=0$

$S=\{\ \}$

12.5 $\dfrac{1}{x-1}-\dfrac{2}{x+1}=-\dfrac{3}{1-x^2}$

$\Leftrightarrow\dfrac{1}{x-1}-\dfrac{2}{x+1}=\dfrac{3}{x^2-1}$

$\Leftrightarrow\dfrac{1}{x-1}-\dfrac{2}{x+1}=\dfrac{3}{(x-1)(x+1)}$

$\Leftrightarrow\dfrac{(x+1)-2(x-1)-3}{x^2-1}\Leftrightarrow\dfrac{x+1-2x+2-3}{x^2-1}$

$\Leftrightarrow\dfrac{-x}{x^2-1}\Leftrightarrow-x=0\wedge x^2-1\neq0\Leftrightarrow x=0$

$S=\{0\}$

12.6 $\frac{1}{3}+\frac{1-2x}{6-3x}=\frac{x^2}{x^2-4}$

$\Leftrightarrow \frac{1}{3}+\frac{1-2x}{3(2-x)}=\frac{x^2}{(x-2)(x+2)}$

$\Leftrightarrow \frac{1}{3}-\frac{1-2x}{3(x-2)}=\frac{x^2}{(x-2)(x+2)}$

$\Leftrightarrow \frac{(x-2)(x+2)}{3(x-2)(x+2)}-\frac{(1-2x)(x+2)}{3(x-2)(x+2)}=\frac{3x^2}{3(x-2)(x+2)}$

$\Leftrightarrow \frac{x^2+2x-2x-4-x-2+2x^2+4x-3x^2}{3(x-2)(x+2)}=0$

$\Leftrightarrow \frac{-4-2+3x}{3(x-2)(x+2)}=0 \Leftrightarrow \frac{3x-6}{3(x-2)(x+2)}=0$

$\Leftrightarrow \frac{3(x-2)}{3(x-2)(x+2)}=0 \Leftrightarrow \frac{1}{(x+2)}=0$

$\Leftrightarrow 1=0 \wedge x+2\neq 0$

$S=\{\ \}$

13.1 $\frac{2}{x-1}-\frac{3}{1-x}=1$

$\Leftrightarrow \frac{2}{x-1}+\frac{3}{x-1}=\frac{x-1}{x-1}$

$\Leftrightarrow \frac{6-x}{x-1}=0$

$\Leftrightarrow 6-x=0 \wedge x-1\neq 0$

$\Leftrightarrow x=6 \wedge x\neq 1$

$S=\{6\}$

13.2 $\frac{x^2}{x+1}-\frac{x}{x+1}=\frac{3}{2}$

$\Leftrightarrow \frac{2(x^2-x)}{2(x+1)}=\frac{3(x+1)}{2(x+1)}$

$\Leftrightarrow \frac{2x^2-2x-3x-3}{2(x+1)}=0$

$\Leftrightarrow \frac{2x^2-5x-3}{2(x+1)}=0$

$\Leftrightarrow 2x^2-5x-3=0 \wedge 2(x+1)\neq 0$

$\Leftrightarrow x=\frac{5\pm\sqrt{25+24}}{4} \wedge x\neq -1$

$\Leftrightarrow \left(x=3 \vee x=-\frac{1}{2}\right) \wedge x\neq -1$

$S=\left\{-\frac{1}{2},3\right\}$

13.3 $\frac{x}{x-1}-\frac{1}{x-1}-\frac{3}{4}=\frac{3}{4x}$

$\Leftrightarrow \frac{(x-1)(4x)}{(x-1)(4x)}-\frac{3x\cdot(x-1)}{(x-1)(4x)}=\frac{3(x-1)}{(x-1)(4x)}$

$\Leftrightarrow \frac{4x^2-4x-3x^2+3x-3x+3}{(x-1)(4x)}=0$

$\Leftrightarrow \frac{x^2-4x+3}{(x-1)(4x)}=0$

$\Leftrightarrow x^2-4x+3=0 \wedge (x-1)(4x)\neq 0$

$\Leftrightarrow x=\frac{4\pm\sqrt{16-12}}{2} \wedge (x\neq 1 \wedge x\neq 0)$

$\Leftrightarrow (x=3 \vee x=1) \wedge (x\neq 1 \wedge x\neq 0) \Leftrightarrow x=3$

$S=\{3\}$

13.4 $\frac{24x}{x^2-16}-\frac{3x}{x-4}=\frac{5}{x+4}$

$\Leftrightarrow \frac{24x}{(x-4)(x+4)}-\frac{3x}{x-4}=\frac{5}{x+4}$

$\Leftrightarrow \frac{24x}{(x-4)(x+4)}-\frac{3x\cdot(x+4)}{(x-4)(x+4)}-\frac{5(x-4)}{(x-4)(x+4)}=0$

$\Leftrightarrow \frac{24x-3x^2-12x-5x+20}{(x-4)(x+4)}=0$

$\Leftrightarrow \frac{-3x^2+7x+20}{(x-4)(x+4)}=0$

$\Leftrightarrow -3x^2+7x+20=0 \wedge (x-4)(x+4)\neq 0$

$\Leftrightarrow \frac{-7\pm\sqrt{49+240}}{-6} \wedge (x-4\neq 0 \wedge x+4\neq 0)$

$\Leftrightarrow \left(x=-\frac{5}{3} \vee x=4\right) \wedge (x\neq 4 \wedge x\neq -4) \Leftrightarrow x=-\frac{5}{3}$

$S=\left\{-\frac{5}{3}\right\}$

14.1 a) $\frac{x}{3}-1>\frac{2x+1}{4} \Leftrightarrow \frac{4x-12-6x-3}{4\times 3}>0$ **Pág. 55**

$\Leftrightarrow \frac{-2x-15}{12}>0 \Leftrightarrow -2x-15>0$

$\Leftrightarrow -2x>15 \Leftrightarrow 2x<-15 \Leftrightarrow x<-\frac{15}{2}$

$S=\left]-\infty,-\frac{15}{2}\right[$

b) $\frac{x+1}{3-x}<0$

Determinação dos zeros:

$\frac{x+1}{3-x}=0$

$\Leftrightarrow x+1=0 \wedge 3-x\neq 0 \Leftrightarrow x=-1 \wedge x\neq 3$

x	$-\infty$	-1		3	$+\infty$
$x+1$	-	0	+	0	+
$3-x$	+	+	+	0	-
$\frac{x+1}{3-x}$	-	0	+	S.S.	-

Então, $\frac{x+1}{3-x}<0 \Leftrightarrow x<-1 \vee x>3$

Logo, $S=\left]-\infty,-1\right[\cup\left]3,+\infty\right[$

c) $\frac{x+1}{x-3}>2 \Leftrightarrow \frac{x+1}{x-3}-\frac{2(x-3)}{x-3}>0$

$\Leftrightarrow \frac{x+1-2x+6}{x-3}>0 \Leftrightarrow \frac{-x+7}{x-3}>0$

Determinação dos zeros:

$\frac{-x+7}{x-3}=0 \Leftrightarrow -x+7=0 \wedge x-3\neq 0$

$\Leftrightarrow x=7 \wedge x\neq 3$

x	$-\infty$	3		7	$+\infty$
$-x+7$	+	+	+	0	-
$x-3$	-	0	+	+	+
$\frac{-x+7}{x-3}$	-	S.S.	+	0	-

Então, $\frac{-x+7}{x-3}>0 \Leftrightarrow x>3 \vee x<7$

Logo, $S=\left]3,7\right[$

d) $\frac{2x+3}{x} \leq 3 \Leftrightarrow \frac{2x+3}{x} \leq \frac{3x}{x}$

$\Leftrightarrow \frac{-x+3}{x} \leq 0$

Determinação dos zeros:

$\frac{-x+3}{x} = 0 \Leftrightarrow -x+3 = 0 \wedge x \neq 0$

$\Leftrightarrow x = 3 \wedge x \neq 0$

x	$-\infty$	0		3	$+\infty$
$-x+3$	+	+	+	0	-
x	-	0	+	+	+
$\frac{-x+3}{x}$	-	S.S.	+	0	-

Então, $\frac{-x+3}{x} \leq 0 \Leftrightarrow x < 0 \vee x \geq 3$

Logo, $S = \left]-\infty, 0\right[\cup \left[3, +\infty\right[$

e) $\frac{3}{2x+3} \geq 0$

Uma vez que $3 > 0$, tem-se que

$\frac{3}{2x+3} \geq 0 \Leftrightarrow 2x+3 > 0 \Leftrightarrow x > -\frac{3}{2}$

$S = \left]-\frac{3}{2}, +\infty\right[$

f) $\frac{x^2+5}{2-3x} < 0$

Uma vez que $\forall x,\ x^2+5 > 0$, tem-se que

$\frac{x^2+5}{2-3x} < 0 \Leftrightarrow 2-3x < 0 \Leftrightarrow -3x < -2$

$\Leftrightarrow 3x > 2 \Leftrightarrow x > \frac{2}{3}$

$S = \left]\frac{2}{3}, +\infty\right[$

g) $\frac{x^2-25}{x^2+25} \geq 0$

Uma vez que $\forall x, x^2+25 > 0$, tem-se que

$\frac{x^2-25}{x^2+25} \geq 0 \Leftrightarrow x^2-25 \geq 0 \Leftrightarrow (x-5)(x+5) \geq 0$

Determinação dos zeros:

$(x-5)(x+5) = 0 \Leftrightarrow x = 5 \vee x = -5$

x	$-\infty$	-5		5	$+\infty$
$x-5$	-	-	-	0	+
$x+5$	-	0	+	+	+
$(x-5)(x+5)$	+	0	-	0	+

Então, $(x-5)(x+5) \geq 0 \Leftrightarrow x \leq -5 \vee x \geq 5$

Logo, $S = \left]-\infty, -5\right] \cup \left[5, +\infty\right[$

h) $\frac{x-1}{2-3x} \geq 0$

Determinação dos zeros:

$\frac{x-1}{2-3x} = 0 \Leftrightarrow x-1 = 0 \wedge 2-3x \neq 0 \Leftrightarrow$

$\Leftrightarrow x = 1 \wedge x \neq \frac{2}{3}$

x	$-\infty$	$\frac{2}{3}$		1	$+\infty$
$x-1$	-	-	-	0	+
$2-3x$	+	0	-	-	-
$\frac{x-1}{2-3x}$	-	S.S.	+	0	-

Então, $\frac{x-1}{2-3x} \geq 0 \Leftrightarrow x > \frac{2}{3} \vee x \leq 1$

Logo, $S = \left]\frac{2}{3}, 1\right]$

i) $\frac{x^2}{(x-3)(4+x)} \geq 0$

Uma vez que $\forall x, x^2 \geq 0$, tem-se que

$\frac{x^2}{(x-3)(4+x)} \geq 0 \Leftrightarrow (x-3)(4+x) > 0$

Determinação dos zeros:

$x^2 = 0 \wedge (x-3)(4+x) \neq 0 \Leftrightarrow x = 0 \wedge (x = 3 \wedge x = -4)$

x	$-\infty$	-4		3	$+\infty$
$x-3$	-	-	-	0	+
$4+x$	-	0	+	+	+
$(x-3)(4+x)$	+	0	-	0	+

Então, $(x-3)(4+x) > 0 \Leftrightarrow x < -4 \vee x > 3$

Logo, $S = \left]-\infty, -4\right[\cup \{0\} \cup \left]3, +\infty\right[$

j) $\frac{1}{x} > x \Leftrightarrow \frac{1}{x} > \frac{x^2}{x} \Leftrightarrow \frac{-x^2+1}{x} > 0$

Determinação dos zeros:

$\frac{-x^2+1}{x} = 0 \Leftrightarrow -x^2+1 = 0 \wedge x \neq 0 \Leftrightarrow$

$\Leftrightarrow x^2 = 1 \wedge x \neq 0 \Leftrightarrow (x = -1 \vee x = 1) \wedge x \neq 0$

x	$-\infty$	-1		0		1	$+\infty$
$-x^2+1$	-	0	+	+	+	0	-
x	-	-	-	0	+	+	+
$\frac{-x^2+1}{x}$	+	0	-	S.S.	+	0	-

Então, $\frac{-x^2+1}{x} > 0 \Leftrightarrow x < -1 \vee 0 < x < 1$

Logo, $S = \left]-\infty, -1\right[\cup \left]0, 1\right[$

14.2 a) $\frac{x}{30-x} = \frac{3}{5} \Leftrightarrow \frac{5x}{30-x} - \frac{3(30-x)}{30-x} = 0$

$\Leftrightarrow \frac{5x-90+3x}{30-x} = 0 \Leftrightarrow \frac{8x-90}{30-x} = 0$

$\Leftrightarrow 8x-90 = 0 \wedge 30-x \neq 0$

$\Leftrightarrow x = \frac{90}{8} \wedge x \neq 30 \Leftrightarrow x = 11{,}25$

A peça mais pequena tem de comprimento 11,25 cm.

b) $\frac{x}{30-x} < \frac{3}{5} \Leftrightarrow \frac{5x}{30-x} - \frac{3(30-x)}{30-x} < 0$

$\Leftrightarrow \frac{5x-90+3x}{30-x} < 0 \Leftrightarrow \frac{8x-90}{30-x} < 0$

Determinação dos zeros:

$8x - 90 = 0 \wedge 30 - x \neq 0 \Leftrightarrow$

$\Leftrightarrow x = \frac{45}{4} \wedge x \neq 30$

x	$-\infty$	$\frac{45}{4}$		30	$+\infty$
$8x$-90	-	0	+	+	+
30-x	+	+	+	0	-
$\frac{8x-90}{30-x}$	-	0	+	S.S.	-

Então, $\frac{8x-90}{30-x} < 0 \Leftrightarrow x < \frac{45}{4} \vee x > 30$

Logo, $S = \left]-\infty, \frac{45}{4}\right[\cup \left]30, +\infty\right[$

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1. $\frac{x^2+4x+3}{x+3} = 0 \Leftrightarrow$

$\Leftrightarrow x^2+4x+3 = 0 \wedge x+3 \neq 0 \Leftrightarrow$

$\Leftrightarrow (x+1)(x+3) = 0 \wedge x+3 \neq 0 \Leftrightarrow$

$\Leftrightarrow (x=-1 \vee x=-3) \wedge x \neq -3 \Leftrightarrow$

$S = \{-1\}$

Resposta: (C)

2. $\frac{1}{4} + \frac{1}{5} = \frac{1}{t}$

Nota: Por lapso, nenhuma das alternativas de resposta está correcta.

3. $\frac{x}{x^2-6x+5} = 0$

$D = \{x \in \mathbb{R} : x^2 - 6x + 5 \neq 0\}$

Tem-se que:

$x^2 - 6x + 5 = 0$

$\Leftrightarrow x = \frac{6 \pm \sqrt{36-20}}{2}$

$\Leftrightarrow x = 5 \vee x = 1$

Logo,

$D = \mathbb{R} \setminus \{1, 5\}$

Resposta: (D)

4. (A) $\frac{x}{x-3} = \frac{3}{3-x}$

$\Leftrightarrow \frac{x}{x-3} = -\frac{3}{x-3} \Leftrightarrow \frac{x+3}{x-3} \Leftrightarrow x-3 \neq 0$

Esta hipótese está excluída.

(B) $\frac{x^3-8}{x^2-4} = 0$

$D = \{x \in \mathbb{R} : x^2 - 4 \neq 0\}$

$D = \mathbb{R} \setminus \{-2, 2\}$

Determinação dos zeros do numerador:

$x^3 - 8 = 0 \Leftrightarrow x = 2$

Uma vez que $2 \notin D$, esta hipótese está excluída.

(D) $\frac{2-x}{x} > 0$

$\Leftrightarrow (2-x > 0 \wedge x > 0) \vee (2-x < 0 \wedge x < 0)$

Esta hipótese está excluída.

(C) Uma vez que $\forall x, x^2+1 > 0$, então,

$\frac{x^3-1}{x^2+1} = 0 \Leftrightarrow x^3 - 1 = 0$

Esta hipótese é verdadeira.

Resposta: (C)

5. Nota: Por lapso, existem três respostas correctas, quando apenas deveria existir uma.

(A) $\frac{3}{x-4} > 0$

Uma vez que $3 > 0$, então,

$\frac{3}{x-4} > 0 \Leftrightarrow x - 4 > 0$

Esta hipótese é verdadeira.

(B) $\frac{-5}{x-8} \geq 0$

Uma vez que -5<0, então,

$\frac{-5}{x-8} \geq 0 \Leftrightarrow x - 8 < 0$

Esta hipótese é falsa.

(C) $\frac{x^2+1}{1-x} > 0$

Uma vez que $\forall x, x^2+1 > 0$, então,

$\frac{x^2+1}{1-x} > 0 \Leftrightarrow 1 - x > 0$

Esta hipótese é verdadeira.

(D) $\frac{x^3}{x^4+1} > 0$

Uma vez que $\forall x, x^4+1 > 0$, então,

$\frac{x^3}{x^4+1} > 0 \Leftrightarrow x^3 > 0 \Leftrightarrow x > 0$

Esta hipótese é verdadeira.

6. $\frac{x^2+3}{x^2}$

$D = \{x \in \mathbb{R} : x^2 \neq 0\}$

Logo,

$D = \mathbb{R} \setminus \{0\}$

Resposta: (B)

7. (A) $(2x+1) = 0 \Leftrightarrow x = -\frac{1}{2}$

Esta hipótese está excluída.

(C) $(x^2-4)(x+1) > 0 \Leftrightarrow$

$\Leftrightarrow (x^2-4 > 0 \wedge x+1 > 0) \vee (x^2-4 < 0 \wedge x+1 < 0)$

Esta hipótese está excluída.

(D) $x^2 \cdot (x-1) \geq 0 \Leftrightarrow$

$\Leftrightarrow x^2 \geq 0 \wedge x - 1 \geq 0$

Esta hipótese está excluída.

(B) $(x^2+4)(x-1) = 0$

Uma vez que $\forall x, x^2+4 > 0$, então,

$(x^2+4)(x-1) = 0 \Leftrightarrow x - 1 = 0$

Esta hipótese é verdadeira.

Resposta: (B)

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8.1 $\frac{x}{x-3}$

$D=\{x\in\mathbb{R}: x-3\neq 0\}$

$D=\{x\in\mathbb{R}: x\neq 3\}$

$D=\mathbb{R}\setminus\{3\}$

8.2 $\frac{1}{x^2-9x}$

$D=\{x\in\mathbb{R}: x^2-9x\neq 0\}$

Determinação dos zeros do denominador:

$x^2-9x=0 \Leftrightarrow x\cdot(x-9)=0$

$\Leftrightarrow x=0 \vee x=9$

Logo,

$D=\mathbb{R}\setminus\{0\,,9\}$

9. $A(x)=\frac{x^2-25}{x^2+5x}$

$D=\{x\in\mathbb{R}: x^2+5x\neq 0\}$

Determinação dos zeros do denominador:

$x^2+5x=0 \Leftrightarrow x\cdot(x+5)=0$

$\Leftrightarrow x=0 \vee x=-5$

Logo,

$D_{A(x)}=\mathbb{R}\setminus\{-5\,,0\}$

$B(x)=\frac{x-5}{x}$

$D=\{x\in\mathbb{R}: x\neq 0\}$

Logo,

$D_{B(x)}=\mathbb{R}\setminus\{0\}$

Então, uma vez que $D_{A(x)}\neq D_{B(x)}$, as expressões $A(x)$ e $B(x)$ não são equivalentes.

10.1 $\frac{4x^2-8}{32-8x^4}=\frac{4\cdot(x^2-2)}{8\cdot(4-x^4)}=$

$=\frac{-4\cdot(2-x^2)}{8\cdot(2-x^2)(2+x^2)}=\frac{-1}{2\cdot(2+x^2)}=\frac{-1}{4+2x^2}$

$D=\{x\in\mathbb{R}: 32-8x^4\neq 0\}$

Determinação dos zeros do denominador:

$32-8x^4=0 \Leftrightarrow x^4=4$

$\Leftrightarrow x=-\sqrt[4]{4} \vee x=\sqrt[4]{4} \Leftrightarrow x=-\sqrt{2} \vee x=\sqrt{2}$

Logo,

$D=\mathbb{R}\setminus\{-\sqrt{2}\,,\sqrt{2}\}$

10.2 $\frac{x^2-5x+6}{x^2-4}=$

$=\frac{(x-3)(x-2)}{(x-2)(x+2)}=$

$=\frac{x-3}{x+2}$

Cálculo auxiliar:

$x^2-5x+6=0 \Leftrightarrow$

$\Leftrightarrow x=\frac{5\pm\sqrt{25-24}}{2} \Leftrightarrow$

$\Leftrightarrow x=3 \vee x=2$

$D=\{x\in\mathbb{R}: x^2-4\neq 0\}$

Determinação dos zeros do denominador:

$x^2-4=0 \Leftrightarrow x^2=4$

$\Leftrightarrow x=-2 \vee x=2$

Logo,

$D=\mathbb{R}\setminus\{-2\,,2\}$

11.1 $\frac{2}{x}+\frac{3}{2x}-\frac{5}{x^2}=\frac{4x+3x-10}{2x^2}=\frac{7x-10}{2x^2}$

$D=\{x\in\mathbb{R}: 2x^2\neq 0\}$

$D=\mathbb{R}\setminus\{0\}$

11.2 $\frac{(x+1)\times 5}{x^2-1}=\frac{(x+1)\times 5}{(x-1)(x+1)}=\frac{5}{(x-1)}$

$D=\{x\in\mathbb{R}: x^2-1\neq 0\}$

Tem-se que:

$x^2-1=0 \Leftrightarrow x=-1 \vee x=1$

$D=\mathbb{R}\setminus\{-1\,,1\}$

11.3 $\frac{x^2-4x+3}{2x^2-x-10}:\frac{x-1}{x+2}=$

$=\frac{x^2-4x+3}{2x^2-x-10}\times\frac{x+2}{x-1}=$

$=\frac{(x-3)(x-1)(x+2)}{2\left(x-\frac{5}{2}\right)(x+2)(x-1)}=$

$=\frac{x-3}{2x-5}$

$D=\{x\in\mathbb{R}: 2x^2-x-10\neq 0 \wedge x-1\neq 0 \wedge x+2\neq 0\}$

Tem-se que:

- zeros de $(2x^2-x-10)$ são $\frac{5}{2}$ e -2;
- zero de $(x-1)$ é 1;
- zero de $(x+2)$ é -2.

$D=\mathbb{R}\setminus\left\{-2\,,1\,,\frac{5}{2}\right\}$

12.1 $\frac{5}{x-1}=0$

Uma vez que $5\neq 0$, então,

$S=\{\ \}$, equação impossível.

12.2 $\frac{1}{x}+\frac{1}{x+4}=\frac{5}{x^2+4x}$

$\Leftrightarrow \frac{(x+4)+x-5}{x^2+4x}=0 \Leftrightarrow \frac{2x-1}{x^2+4x}=0$

$\Leftrightarrow 2x-1=0 \wedge x^2+4x\neq 0$

$\Leftrightarrow x=\frac{1}{2} \wedge x\cdot(x+4)\neq 0$

$\Leftrightarrow x=\frac{1}{2} \wedge (x\neq 0 \wedge x\neq -4)$

Logo,

$S=\left\{\frac{1}{2}\right\}$ e $D=\mathbb{R}\setminus\{-4\,,0\}$

13.1 $\frac{3}{2x+1}\leq 0$

Uma vez que $3>0$, então,

$\frac{3}{2x+1}\leq 0 \Leftrightarrow 2x+1<0 \Leftrightarrow x<-\frac{1}{2}$

$S=\left]-\infty\,,-\frac{1}{2}\right[$

13.2 $\frac{1}{3x+1}\geq\frac{1}{4} \Leftrightarrow \frac{4-(3x+1)}{4\cdot(3x+1)}\geq 0$

$\Leftrightarrow \frac{3-3x}{12x+4}\geq 0$

Determinação dos zeros:

- $3-3x=0 \Leftrightarrow x=1$
- $12x+4=0 \Leftrightarrow x=-\frac{1}{3}$

x	$-\infty$	$-\frac{1}{3}$		1	$+\infty$
3-3x	+	+	+	0	-
12x+4	-	0	+	+	+
$\frac{3-3x}{12x+4}$	-	S.S.	+	0	-

Então, $\frac{3-3x}{12x+4} \geq 0 \Leftrightarrow -\frac{1}{3} < x \leq 1$

Logo, $S=\left]-\frac{1}{3},1\right]$

Nota : Por lapso, a solução apresentada no manual não está correcta.

14. $240=(y-2\times 2)(x-2\times 1{,}5)$

$\Leftrightarrow 240=(y-4)(x-3)$

$\Leftrightarrow 240=y\cdot x-3y-4x+12$

$\Leftrightarrow y(x-3)=228+4x$

$\Leftrightarrow y=\frac{228+4x}{x-3}$

Por sua vez, $A(x)=x\cdot y$

Substituindo y, vem:

$A(x)=x\cdot\frac{228+4x}{x-3} \Leftrightarrow$

$\Leftrightarrow A(x)=x\cdot\frac{228x+4x^2}{x-3}$ c.q.d

15. Área do círculo de diâmetro $\overline{AB}$:

$\pi\left(\frac{40}{2}\right)^2=400\pi$

Área do círculo de diâmetro $\overline{AC}$:

$\pi\left(\frac{40-x}{2}\right)^2=\frac{1600-80x+x^2}{4}\pi$

Área do círculo de diâmetro $\overline{BC}$:

$\pi\left(\frac{x}{2}\right)^2=\frac{x^2}{4}\pi$

Então, a área da zona relvada (A) obtém-se do seguinte modo:

$A=400\pi-\frac{1600-80x+x^2}{4}\pi-\frac{x^2}{4}\pi$

$\Leftrightarrow A=400\pi-400\pi+20\pi\cdot x-\frac{2x^2}{4}\pi$

$\Leftrightarrow A=20\pi\cdot x-\frac{x^2}{2}\pi$

$\Leftrightarrow A=\frac{\pi}{2}\left(-x^2+40x\right)$ c.q.d.